

Time: 3 Hours

Marks: 80

Note : 1) Q.1 is COMPULSORY.

2) Attempt ANY 3 questions from Q.2 to Q.6

3) Use of scientific calculators allowed.

4) Figures to right indicate marks.

Q.1 a) Find the Laplace transform of  $t e^t \sin 2t \cos t$ . (05)b) Find the inverse Laplace transform of  $\frac{s+2}{s^2(s+3)}$  (05)c) Determine whether the function  $f(z) = x^2 - y^2 + 2ixy$  is analytic and if so find its derivative. (05)d) Find the Fourier series for  $f(x) = e^{-|x|}$  in the interval  $(-\pi, \pi)$ . (05)Q.2 a) Evaluate  $\int_0^\infty \frac{e^{-t} - \cos t}{te^{4t}} dt$  (06)b) Find the Z- Transform of  $f(k) = \begin{cases} 3^k, & k < 0 \\ 2^k, & k \geq 0 \end{cases}$  (06)c) Show that the function  $u = 2x(1 - y)$  is a harmonic function. Find its harmonic conjugate and corresponding analytic function. (08)

Q.3 a) Find the equation of the line of regression of y on x for the following data (06)

X	10	12	13	16	17	20	25
y	19	22	24	27	29	33	37

b) Find the bilinear transformation which maps  $z = 2, 1, 0$  onto  $w = 1, 0, i$ . (06)c) Obtain the expansion of  $f(x) = x(\pi - x)$ ,  $0 < x < \pi$  as a half range cosine series.Hence show that  $\sum_{n=1}^\infty \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$ . (08)

Q.4 a) Find the inverse Laplace Transform by using convolution theorem

$$\frac{1}{(s^2 + 1)(s^2 + 9)} \quad (06)$$

b) Calculate the coefficient of correlation between Price and Demand. (06)

Price : 2, 3, 4, 7, 4.

Demand : 8, 7, 3, 1, 1.

c) Find the inverse Z-transform for the following ;

(08)

i)  $\frac{z}{z-5}$  ,  $|z| < 5$

ii)  $\frac{1}{(z-1)^2}$  ,  $|z| > 1$

Q.5 a) Find the Laplace transform of  $e^{-t} \sin t H(t - \pi)$

(06)

b) Show that the set of functions  $\{\sin x, \sin 3x, \sin 5x, \dots\}$  is orthogonal over  $[0, \pi/2]$ . Hence construct orthonormal set of functions.

(06)

c) Solve using Laplace transform  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3t e^{-t}$ ,  
given  $y(0) = 4$  and  $y'(0) = 2$ .

(08)

Q.6 a) Find the complex form of Fourier series for  $f(x) = 3x$  in  $(0, 2\pi)$ .

(06)

b) If  $f(z)$  is an analytic function with constant modulus then ,  
prove that  $f(z)$  is constant.

(06)

c) Fit a curve of the form  $y = ax^b$  to the following data.

(08)

x	1	2	3	4
y	2.5	8	19	50

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