Total Marks: 80   **Time Duration: 3Hr**

N.B.:1) Question no.1 is compulsory.   **Maximum**
   2) Attempt any three questions from Q.2to Q.6.
   3) Figures to the right indicate full marks.   **Marks**

Q1. a) Find the Laplace transform of $\cos 2t \sin t\, e^{-t}$.   **[5]**

b) Find the half-range sine series for $f(x) = x(\pi - x)$ in $(0, \pi)$.   **[5]**

c) Show that the function $f(z) = ze^z$ is analytic and find $f'(z)$ in terms of z.   **[5]**

b) Prove that $\nabla\left\{\nabla.\dfrac{\bar{r}}{r}\right\} = -\dfrac{2}{r^3}\bar{r}$.   **[5]**

Q2. a) Find the inverse Z-transform of $F(z) = \dfrac{z}{(z-1)(z-2)}$ $|z| > 2$.   **[6]**

b) Find the analytic function whose real part is $\dfrac{\sin 2x}{\cosh 2y + \cos 2x}$.   **[6]**

c) Obtain Fourier series for the function $f(x) = \begin{cases} 1 + \dfrac{2x}{\pi} & , -\pi \le x \le 0 \\ 1 - \dfrac{2x}{\pi} & , 0 \le x \le \pi \end{cases}$,   **[8]**

deduce that $\dfrac{\pi^2}{8} = \dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \cdots$

Q3. a) Find $L^{-1}\left[\dfrac{1}{s^2(s+a)^2}\right]$ using convolution theorem.   **[6]**

b) Show that the set of functions $\cos nx$, $n = 1, 2, 3 \ldots$ is orthogonal on $[0, 2\pi]$.   **[6]**

c) Using Green's theorem evaluate $\int_c \left(\dfrac{1}{y} dx + \dfrac{1}{x} dy\right)$ where C is the boundary of the   **[8]**

region defined by $x = 1, x = 4, y = 1$ and $y = \sqrt{x}$.

Q4. a) Find Laplace transform of $f(t) = k\dfrac{t}{T}$ for $0 < t < T$ and $f(t) = f(t + T)$.   **[6]**

b) Show that $\bar{f} = (x^2 + xy^2)\,i + (y^2 + x^2y)\,j$ is irrotational and find its scalar potential.   **[6]**

c) Find half – range cosine series for $f(x) = x$, $0 < x < 2$. Using Parseval's identity   **[8]**
deduce that
i) $\dfrac{\pi^4}{96} = \dfrac{1}{1^4} + \dfrac{1}{3^4} - \dfrac{1}{5^4} + \cdots$
ii) $\dfrac{\pi^4}{90} = \dfrac{1}{1^4} + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \cdots$

Q5.a) Use divergence theorem to show that $\iint_S \nabla r^2\,\overline{ds} = 6v$ where S is any closed surface   **[6]**
enclosing a volume V.

b) Find the Z-transform of $f(k) = k\alpha^k$, k$\ge$ 0 .   **[6]**

c) i) Find $L^{-1}\left[\dfrac{(s+2)^2}{(s^2+4s+8)^2}\right]$ **[8]**
ii) Find $L^{-1}[2\tanh^{-1} s]$

Q6.a) Solve using Laplace transform   **[6]**
$(D^2 - 3D + 2)y = 4e^{2t}$, with $y(0) = -3, y'(0) = 5$.

b) Find the bilinear transformation which maps the points 1, -i, 2 on z-plane onto 0, 2, -i   **[6]**
respectively of w-plane.

c) Express the function $f(x) = \begin{cases} \sin x & , 0 < x \le \pi \\ 0 & x < 0, x > \pi \end{cases}$ as Fourier integral and deduce   **[8]**

that $\int_0^\infty \dfrac{\cos\left(\frac{w\pi}{2}\right)}{1-w^2}\, dw = \dfrac{\pi}{2}$.

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