Total Marks:80

Duration: 3Hrs

NB 1. Question No.I is compulsory.
2. Attempt any **three** from the remaining six questions.
3. Figures to the right indicate full marks.

Q1a. If $L\{tsin\omega t\} = \frac{2\omega s}{(s^2+\omega^2)^2}$, find $L\{\omega t cos\omega t + sin\omega t\}$ [20]

b. If $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic ,find a,b,c and d

c. **Find** the Fourier series expansion of $f(x) = x^3 \ \ (-\pi, \pi)$

d. If the two regression equations are $4x - 5y + 33 = 0, 20x - 9y - 107 = 0$ find i) the mean values of *x* and *y*

ii) the Correlation Coefficient iii) Standard deviation of *y* if variance of x is 9

Q2 a. Show that the function is Harmonic and find the Harmonic Conjugate
$u = cosx\ coshy - 2xy$ [6]

b. Evaluate $\int_0^\infty e^{-t}(\int_0^t u^2\ sinhu\ coshu\ du)dt$) using Laplace Transform. [6]

c. Find Fourier Series expansion of $f(x) = x \quad -1 < x < 0$
$= x + 2 \quad 0 < x < 1$ [8]

Q3 a. Find the Analytic function $f(z) = u + iv$ if $u - v = e^x(cosy - siny)$ [6]

b. Find Inverse Z transform of $\frac{5z}{(2z-1)(z-3)} \quad \frac{1}{2} < |z| < 3$ [6]

c. Solve the Differential Equation using Laplace transform
$(D^2 - 2D + 1)y = e^t$, $y(0) = 2$, y'(0)= -1 [8]

Q4 a. Find the Complex Form of Fourier Series for $f(x) = cosax \ \ (-\pi, \pi)$ [6]

b. Find the Spearman's Rank correlation coefficient between *X* and *Y*. [6]

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

c. Find the inverse Laplace transform of i)$\frac{s-1}{s^2+2s+2}$ ii)$\frac{e^{-\pi s}}{s^2(s^2+1)}$ [8]

Q5 a. Find the $Z\{f(k)\}$ where $f(k) = 4^k\,, k < 0$ [6]
$=3^k\,,\ k \geq 0$

b. Show that $\{cosx\,, cos2x, cos3x, \ldots\ldots\}$ is orthogonal set over the interval $[0,2\pi]$. Construct the corresponding orthonormal set. [6]

c. Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$, Hence find the image of $|z| < 1$. [8]

Q6 a. Fit a straight line to the given data [6]

X	10	12	15	23	20
Y	14	17	23	25	21

b. Find Inverse Laplace Transform using Convolution theorem $\frac{1}{(s-2)^3(s+3)}$ [6]

c. Find Half Range Cosine Series for $f(x) = sinx$ in $(0,\pi)$ and hence deduce that
$\frac{\pi^2-8}{16} = \frac{1}{1^2.3^2} + \frac{1}{3^2.5^2} + \frac{1}{5^2.7^2} + \cdots \ldots \ldots$ [8]

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