

Time Duration: 3Hr

Total Marks: 80

N.B.:1) Question no.1 is compulsory.

2) Attempt any three questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

- Q1. a)** Find the Laplace transform of  $e^{-4t}t \sin 3t$ . [5]  
**b)** Find the half-range cosine series for  $f(x) = x$ ,  $0 < x < 2$ . [5]  
**c)** Find  $\nabla \cdot \left( r \nabla \frac{1}{r^3} \right)$ . [5]  
**d)** Show that the function  $f(z) = \sin z$  is analytic and find  $f'(z)$  in terms of  $z$ . [5]
- Q2. a)** Find the inverse Z-transform of  $F(z) = \frac{1}{(z-5)^3}$ ,  $|z| < 5$ . [6]  
**b)** Find the analytic function whose imaginary part is  $e^{-x}(y \sin y + x \cos y)$ . [6]  
**c)** Obtain Fourier series for the function  $f(x) = x + x^2$ ,  $-\pi \leq x \leq \pi$  and  $f(x + 2\pi) = f(x)$ . [8]  
Hence deduce that  $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$  and  $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$
- Q3. a)** Find  $L^{-1} \left[ \frac{1}{(s-a)(s-b)} \right]$  using convolution theorem. [6]  
**b)** Is  $S = \left\{ \sin \left( \frac{\pi x}{4} \right), \sin \left( \frac{3\pi x}{4} \right), \sin \left( \frac{5\pi x}{4} \right), \dots \right\}$  orthogonal in  $(0, 1)$ ? [6]  
**c)** Using Green's theorem in the plane evaluate  $\int_C (xy + y^2)dx + (x^2)dy$  where  $C$  is the closed curve of the region bounded by  $y = x$  and  $y = x^2$ . [8]
- Q4. a)** Find Laplace transform of  $f(t) = \begin{cases} \sin 2t & , 0 < t \leq \frac{\pi}{2} \\ 0 & , \frac{\pi}{2} < t < \pi \end{cases}$  and  $f(t) = f(t + \pi)$ . [6]  
**b)** Prove that a vector field  $\vec{f}$  is irrotational and hence find its scalar potential  $\vec{f} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$ . [6]  
**c)** Find the Fourier expansion for  $f(x) = \sqrt{1 - \cos x}$  in  $(0, 2\pi)$ . Hence deduce that  $\frac{1}{2} = \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$ . [8]
- Q5. a)** Use Gauss's Divergence Theorem to show that  $\iint_S \nabla r^2 \cdot \vec{ds} = 6V$  where  $S$  is any closed surface enclosing a volume  $V$ . [6]  
**b)** Find the Z-transform of  $f(k) = b^k$ ,  $k < 0$ . [6]  
**c)** i) Find  $L^{-1} \left[ \frac{s}{(s-2)^6} \right]$ . [8]  
ii) Find  $L^{-1} \left[ \log \left( 1 + \frac{a^2}{s^2} \right) \right]$ .
- Q6. a)** Solve using Laplace transform  $(D^2 + 9)y = 18t$ , given that  $y(0) = 0$  and  $y \left( \frac{\pi}{2} \right) = 0$  [6]  
**b)** Find the bilinear transformation which maps the points  $Z = \infty, i, 0$  onto  $W = 0, i, \infty$ . [6]  
**c)** Find Fourier integral representation of  $f(x) = e^{-|x|}$   $-\infty < x < \infty$ . [8]