Duration: 3 Hours Marks : 100

N.B. 1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Use of calculator is allowed.

Q.1] a) Define the following terms with suitable example [10]
i. Simple Hypothesis
ii. Null Hypothesis
iii. Critical region
iv. Type I error
v. Power of the test

b) State and prove Neyman-Pearson Lemma for obtaining the best test for testing simple Null hypothesis against simple alternative hypothesis. [10]

OR

p) **I)** A Random variable X has the following p.d.f [05]

$$f(x) = \begin{cases} ½ , & \theta - 1 \leq x \leq \theta + 1 \\ 0 & \text{otherwise} \end{cases}$$

to test the hypothesis H_0: θ = 4 against H_1: θ = 5. Determine the value of 'c' if the critical region is given by x >c. α=0.025. Also calculate the power of test.

II) If x ≥ 1 is the critical region for testing H_0: θ =2 against H_1: θ =1 based on a single observation from a population with p.d.f [05]

$$f(x,\theta) = \begin{cases} \theta e^{-\theta x}; & x \geq 0, \theta > 0 \\ 0, & O.W. \end{cases}$$

Calculate size and power of the test.

q) Derive the Most powerful test of size α for testing H_0 : θ = θ_0 against H_1 : θ = θ_1 when a sample of size n is taken from Bernoulli (θ) for the following cases. [10]
Case (i) $\theta_0 < \theta_1$
Case (ii) $\theta_0 > \theta_1$

Q.2] a) Define Uniformly Most Powerful Test (UMPT). Obtain UMPT of size α to test H_0 : θ = θ_0 against H_1 : θ > θ_1, where $X_1, X_2, \ldots, X_n$ is a random sample of size n drawn from N(0,θ). [10]

b) I) Define Likelihood Ratio Test (LRT) and write down its procedure. (07)

II) Comment: Level of significance is always equal to the probability of type I error. (03)

OR

p) Obtain UMPT of size α to test H_0 : θ = θ_0 against H_1 : θ = θ_1, where $X_1, X_2, \ldots, X_n$ is a random sample of size n drawn from Exponential (mean= $\frac{1}{\theta}$). [10]
Case I) $\theta_1 > \theta_0$ II) $\theta_1 < \theta_0$

q) Let $X_1, X_2, \ldots, X_n$ be a random sample drawn from N(μ, σ^2) population where σ^2 is known. Derive LRT to test H_0 : $\mu = \mu_0$ against H_1 : $\mu \neq \mu_0$. [10]

Q.3] a) Discuss the test procedure of Sequential Probability Ratio Test (SPRT) for testing a simple null hypothesis against a simple alternative hypothesis. [10]
Differentiate between Sequential test procedures with Neyman Pearson's test procedure.

b) Construct SPRT of strength (α, β) for testing $H_0:\theta=\theta_0$ against $H_1:\theta=\theta_1$ $(\theta_1 < \theta_0)$, when a random variable X follows Binomial distribution with parameters (k, θ). [10]

OR

p) A random variable X follows a Poisson distribution with parameter λ Derive SPRT of strength (α, β) to test the hypothesis $H_0: \lambda = \lambda_0$ v/s $H_1: \lambda = \lambda_1$, $(\lambda_0 > \lambda_1)$. [10]

q) Construct SPRT of strength (α, β) for testing $H_0:\mu = \mu_0$ against $H_1: \mu = \mu_1$, $(\mu_0 > \mu_1)$ from $N(\mu, \sigma^2)$, where σ^2 is known. [10]

Q.4] a) Stating the assumption clearly, explain the Sign test for single sample. Also explain the Normal approximation for sign test. [12]

b) Explain the median test for two independent samples. [08]

OR

p) I) Explain Wilcoxon matched pair signed rank test for two related samples. (05)
II) Explain the Wald-Wolfowitz Run test. (05)

q) Explain Kruskal Wallis one way ANOVA by Ranks test stating all the assumptions. [10]

Q.5] Attempt **Any Two** sub questions

a) Define the following terms with suitable example [10]
1. Alternative hypothesis
2. Type II error
3. P-value
4. One sided test
5. Acceptance region

b) Let $X_1, X_2, \ldots, X_n$ be a random sample drawn from $N(\mu, \sigma^2)$ population where μ is known. Derive LRT to test $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$. [10]

c) A random variable X follows an Exponential distribution with unknown parameter θ. Derive SPRT of strength (α, β) for testing $H_0 : \theta=\theta_0$ against $H_1:\theta=\theta_1$ $(\theta_1 < \theta_0)$. [10]

d) Explain Friedman Two way ANOVA by ranks test stating the assumptions [10]

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