N.B.(i) All questions are compulsory.
(ii) Figures to the right indicate full marks.

Time : 3 Hours **Marks : 100**

Q.1 (a) Suppose (X, Y) follows Bivariate Normal Distribution with parameters $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, derive marginal distribution of X and marginal distribution of Y. **(10)**

(b) Suppose (X, Y) follows Bivariate Normal Distribution with parameters $(0, 0, 1, 1, \rho)$. Obtain its joint MGF and hence marginal distributions of X and Y. **(10)**

OR

(p) Define fisher's Z transformation and explain its use in testing **(10)**

i. $H_0: \rho = \rho_0$ against $H_1: \rho > \rho_0$
ii. $H_0: \rho = \rho_0$ against $H_1: \rho < \rho_0$
iii. $H_0: \rho = \rho_0$ against $H_1: \rho \neq \rho_0$ for large samples, in case of bivariate normal population.

Also obtain 95% confidence interval for ρ.

(q) Let X, Y and Z be i.i.d. Normal(0,1). **(10)**

i. Show that Joint distribution of U = X + aZ and V = Y + aZ is Bivariate Normal.
ii. In above example, find 'a' such that correlation coefficient between U and V is $\frac{1}{2}$

Q.2 **(a)** Let X be a discrete r.v. such that $P(X=n) = P_n$ n = 0,1,2………. **(10)**
Obtain generating functions of the following sequences.

i. $q_n = p(X \geq n)$
ii. $q_n = p(X < n)$

(b) i. Define Probability Generating Function (PGF) of discrete random variable assuming integral values **(02)**
ii) Obtain PGF of Negative Binomial distribution and hence derive its mean and variance. **(08)**

OR

(p) i. A uniform die is thrown. Let X denote number on uppermost face of a die. Find PGF of X. **(04)**
ii. Bernoulli trial is repeated till success is occurred K times. Let X be the number of tra=ilas required to get k^{th} success. Obtain PGF of X and mean. **(06)**

(q) i. Let $x_1, x_2, \ldots, x_n$ be n independent random variables with PGFs $P_1(S)$, $P_2(S)$, …, $P_n(S)$ respectively. Obtain the PGF of $\sum x_i$. **(10)**
ii. Let U(S) and V(S) be the generating functions of two independent random variables, respectively. Obtain the generating functions of (a) X + Y and (b) X – Y

Q.3 **(a)** In the usual notations, for Poisson birth process, list the postulates, state the expression $P_n(t)$, the probability of 'n' numbers in the system at time 't' and find its mean and variance. **(10)**

(b) Stating clearly postulates for the pure death process, with initially 'i' members in the system at time t=0. Derive the difference differential equation for $\mu_n = n\mu$ and find the expression for the probability of 'n' numbers in the system at time 't'. **(10)**

OR

(p) For linear growth model having birth rate $(n\lambda)$ and death rate $(n\mu)$ and initially at time $t = 0$, there are 'a' member in the system then **(10)**

i. List the Postulates.
ii. Derive difference differential equation.

(q) In usual notation for Poisson death process, list the postulates, state the expression for $P_n(t)$, probability that 'n' members in the system at time 't'. Find its mean and variance. **(10)**

Q.4 **(a)** Define the following terms; **(10)**

(i) Input Process
(ii) Server and customer
(iii) Reneging
(iv) Jockeying
(v) Balking

(b) Show that for a single service station with Poisson arrival and exponential service time the probability that exactly 'n' calling units are in the queue system is, **(10)**

$$P_n = (1 - \rho)\rho^n \quad ; n \geq 0$$

Where, ρ is the traffic intensity.

OR

(p) For ***{(M/M/1) : (N/FIFO)}*** queueing model, derive the expression for P_n and find *E(n).* **(10)**

(q) Discuss the classification of queuing model, and operating characteristics of queueing system. **(10)**

Q.5 Attempt **Any Two** sub questions

(a) Show that (X, Y) follows Bivariate Normal Distribution with parameters $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ if and only if every linear combination of X and Y viz. ax+by, a≠0 , b≠0, is a Normal Variate. Where a and b are constants. **(10)**

(b) Let X be a r.v. assuming values 0, 1, 2, …. with probabilities p_0, p_1, p_2, ….. Let qj = P(X > j) and pj = P(X = j), j = 1, 1, 2, …. Assuming actual notations, show that $Q(S) = \frac{1-P(S)}{1-S}$ where $Q(S) = \sum_{j=0}^{\infty} q_j S^j$ and $P(S) = \sum_{j=0}^{\infty} p_j S^j$. Also obtain mean and variance of X in terms of Q. **(10)**

(c) For pure birth process, if the growth rate λ_n is directly proportional to the number of individuals (n ≥ 1) say $\lambda_n \propto n$, assuming no death or removal and initially at time t=0, there are 'i' members in the system, then **(10)**

i. Obtain difference differential equation.

ii. Find $P_n(t)$

(d) What are the service disciplines? Describe some forms of common service Disciplines and illustrate with examples. **(10)**