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Time: 3 hrs. **M. M.: 100**

N.B.:
1. **All** questions are **compulsory**.
2. **Figures** to the **right** indicate **full** marks.
3. Draw **neat** diagrams wherever **necessary**.
4. Symbols have usual meaning unless otherwise stated.
5. Use of **non-programmable** calculator is allowed.

Q1. Attempt any **two**

(i) State and prove Kepler's laws of planetary motion. **10**

(ii) Discuss quantitatively the motion of a particle in an inverse square field. Show that the eccentricity of the particle is given by $\epsilon = \sqrt{1 + \frac{2EL^2}{mK^2}}$. Give conditions on E and ϵ for different shapes of orbits. **10**

(iii) What is a Foucault pendulum? Obtain equation of motion for it. Hence show that the pendulum precesses slowly clockwise in the northern hemisphere. **10**

(iv) Consider a starred (rotating) reference frame rotating with angular velocity ω relative to the unstarred (fixed) frame with their origins O and O^* coinciding. Prove that for an arbitrary vector $\vec{A}$, **10**

$$\frac{d^2\vec{A}}{dt^2} = \frac{d^{*2}\vec{A}}{dt^2} + \vec{\omega} \times (\vec{\omega} \times \vec{A}) + 2\vec{\omega} \times \frac{d^*\vec{A}}{dt} + \frac{d^*\vec{\omega}}{dt} \times \vec{A}$$

Q2 Attempt any **two**

(i) Starting with D'Alembert's Principle, obtain Lagrange's equations in terms of generalized coordinates. **10**

(ii) What is meant by generalized co-ordinates? Derive an expression for generalized velocity and generalized kinetic energy. **10**

(iii) A body of mass m_1 can move on a smooth flat horizontal table top. It is connected to a string of length ℓ which passes through a hole in the centre of the table. The other end of the string is connected to a mass m_2 which is suspended vertically. Identify appropriate generalized coordinate for the system and obtain the equations of motion using D'Alembert's principle. **10**

(iv) A double pendulum consists of two weightless rods connected to each other and a point of support. The masses m_1 and m_2 are not equal but the length of the rods are equal. Pendulums are free to swing only in one vertical plane. Write down the Lagrangian for the system. **10**

Q3 Attempt any **two**

(i) For a moving fluid show that **10**

a) $\frac{dp}{dt} = \frac{\partial p}{\partial t} + \bar{v} \cdot \bar{\nabla} p$

b) $\frac{d}{dt}\ \delta V = \bar{\nabla} \cdot \bar{v}\ \delta V$

(Symbols have their usual meanings)

(ii) Derive Bernouilis theorem. Hence for a steady flow of the fluid show that, **10**

$\frac{v^2}{2} + \frac{p}{\rho} - \mathcal{G} + u = constant$ (Symbols have their usual meanings)

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(iii) With reference to rotations of rigid body explain setting of the Euler's angles. Draw suitable diagrams. Find expression for the Lagrangian of a heavy symmetric top. **10**

(iv) Derive an expression for the moment of inertia tensor for a rigid body made up of N number of particles. **10**

Q4 Attempt any **two**

(i) Discuss numerical solutions of Duffing's equation for $\gamma = 0.1$ *and* $f = 0.5$ and 2) $\gamma = 0.1$ *and* $f = 3$. Compare the nature of odd and even harmonics. **10**

(ii) Consider an anharmonic oscillator with potential energy $V(x) = K\left(\frac{x^2}{2} + \frac{\alpha x^4}{4}\right)$ where K is the spring constant and α is anharmonic coefficient. Discuss the potential energy curve for positive and negative values of K and α. Comment on confinement of motion. **10**

(iii) Discuss fixed points of a logistic map, stability of fixed points and periodic attractors. Discuss logistic map for 3< λ <4 and explain the onset of chaos qualitatively. **10**

(iv) Discuss numerical solutions of Duffing's equation for 1) $\gamma = 0.1$ *and* $f = 20$ and 2) $\gamma = 0.1$ *and* $f = 25$ **10**

Q5. Attempt any **four**

(i) If a body of mass 100 kg is moving with a velocity of 10m/s; estimate the maximum Coriolis force experienced by the body. **05**

(ii) The eccentricity of a planet's orbit about sun is 0.4. Find the ratio of the lengths of the semi major to the semi minor axes of the orbit of the planet. **05**

(iii) Write down the Lagrangian for a simple pendulum and hence find its equation of motion. **05**

(iv) Define constraints. With good examples, explain holonamic and non-holonomic constraints. **05**

(v) Consider a fluid flow given by $\vec{v} = cy\hat{\imath}$. Show that the fluid is incompressible and non-irrotational. **05**

(vi) What is a rigid body? Discuss the different types of rigid bodies with reference to the symmetry present in the body. **05**

(vii) Two very close initial values of x on logistic map are 0.40000 and 0.40002 respectively. With λ=4, after 20 iterations the values are 0.14561 and 0.00170 respectively. Calculate Lyapunov exponent. **05**

(viii) Discuss the nature of phase space diagram for 1)undamped oscillator and 2)damped oscillator. **05**

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