T.Y.B.Sc. (Sem VI) March 2015

Physics

Classical Mechanics

QP Code : 14655

(2½ Hours) [ Total Marks : 75

N. B. : (1) **All** questions are **compulsory.**
(2) **Figures** to the **right** indicate **full** marks.
(3) Symbols have usual meaning unless otherwise stated.
(4) Draw neat diagrams wherever necessary.
(5) Use of logtable or non programmable calculator is allowed.

1. (a) Attempt any **one** :–
(i) Write the equation of motion of a particle in a central force field. Solve it to get equation of orbit, for inverse square law force field. Explain different orbits. 10
(ii) State Coriolis theorem. Give interpretation of various terms in the equation. Explain the effect of centrifugal force with respect to rotating earth. 10
(b) Attempt any **one** :–
(i) State Kepler's laws of planetary motion. Prove that the areal velocity of planet in a orbit is constant. 5
(ii) Explain the effect of Coriolis force on a particle moving in a horizontal plane in the northern hemisphere near the earth surface. 5

2. (a) Attempt any **one** :–
(i) Derive Lagrange's equation of motion in several diamensions with no constraints imposed. 10
(ii) Explain how the forces of constraints are determined used Lagrangian formulation. Illustrate the same taking Atwood's machine as an example. 10
(b) Attempt any **one** :–
(i) Set up the Lagrangian for a simple pendulum and obtain an equation describing its motion without constraint. 5
(ii) Describe generalised co-ordinates. 5

3. (a) Attempt any **one** :–
(i) Derive Bernouli's theorem and discuss how it represents the conservation law of energy for a fluid. 10



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(ii) A rigid body is constrained to move about a point which is fixed.Derive an expression for its angular mementurm about the instantaneous axis of rotation passing through the fixed point. Hence derive an expression for kinctic energy of the body and express in terms of angular momentum. 10

(b) Attempt any **one** :–

(i) In case of a fluid moving with velocity $\vec{v}$, show that 5

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}$$

Using this, show that

$$\frac{d}{dt}(\delta V) = (\vec{\nabla} \cdot \vec{v})\delta V$$

(ii) The Lagrangian for the top is given by 5

$$L = \frac{1}{2} I_1(\dot{\theta}^2 + \dot{\phi}^2 \sin^2 \theta) + \tfrac{1}{2} I_3(\dot{\Psi} + \dot{\phi}\cos\theta)^2 - mgl\cos\theta$$

Find $\phi$ equation of motion of the top.

(a) Attempt any **one** :–

(i) What do you mean by fractals dimension? Show the construction of cantor set and Sierpinski gasket. Hence find fractal dimension in each case. 10

(ii) The reduced Duffing's equation of motion for an anharmonic oscillator is $\ddot{x} + 2\gamma\dot{x} + x \pm x^3 = f\cos\omega t$. Discuss its numerical solution considering $\gamma$ = 0.1 with special reference to sub harmonic resonances and hysteresis for f=3; period - 2 orbits for f=20 and leading to chaos for f=25. 10

(b) Attempt any **one** :–

(i) Show graphical illustration of potential energy of an anharmonic oscillator constructed using hard spring and subjected to a conservative force $F(x) = -k(x + \alpha x^3)$ for $k = \pm 1$. Comment on the motion in each case. 5

(ii) Explain scale invariance in the Poincare section having bifurcations leading to chaos. Hence explain the Feigenbaum number. 5

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(a) Attempt any **one** :–

(i) The eccentricity of planet orbit about the sun is 0.6. Find the ratio of lengths of major to minor axes. 4

(ii) Calculate the time taken by the plane of oscillations of a pendulum at 60° colatitude to turn through 180° in the northern hemisphere. 4

(b) Attempt any **one** :–

(i) Write a short note on constraints. 4

(ii) Write a short note on constants of motion and ignorable co-ordinates. 4

(c) Attempt any **one** :–

(i) Consider a fluid flow in which the velocity is given by $\vec{v}(x,t)=\frac{at}{x}\hat{i} \quad x>0$. Find the acceleration a(x,t) of the fluid element at position x and time t. Is the flow incompressible? Explain. 4

(ii) For a rigid body, the inertia tensor is given by 4

$$\vec{I}=\begin{pmatrix} \frac{1}{3}ma^2 & -\frac{1}{4}ma^2 & 0 \\ -\frac{1}{4}ma^2 & \frac{1}{3}ma^2 & 0 \\ 0 & 0 & \frac{2}{3}ma^2 \end{pmatrix}$$

Find the principal moment of inertia for the body.

(d) Attempt any **one** :–

(i) Write a note on Henon map. 3

(ii) Discuss the chaotic behavior of Lyapunov exponent. 3

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