TYB.Sc (VI) 10/04/15

Algebra - II

Q.P. Code : 14586

(2½ Hours) **[Total Marks : 75**

N.B.: (1) **All** questions are **compulsory**.
(2) Figures to the right indicate full marks.
(3) Use of log table/non programmable calculator is allowed.
(4) Symbols have their usual meanings unless stated otherwise
(5) Draw **neat** diagrams wherever **necessary**.

1. (a) Attempt any one.
   (i) Obtain the Lorentz transformation equations for space time coordinates of an event. State clearly where the postulates of the special theory of relativity have been used in the derivation. **10**
   (ii) State the postulates of the special theory of relativity. Write the Lorentz transformation equations for space - time coordinates of an event (derivation not required). Hence derive the formulae for time dilation and length contraction. **10**

   (b) Attempt any one.
   (i) Assuming the Lorentz transformation equations for space time coordinates of an event, derive the transformation equation for the component of acceleration parallel to the direction of relative motion. **5**
   (ii) Give the wavelength shift, if any, in the Doppler effect for the sodium D_2 line ($5890\ \overset{0}{A}$) emitted from a source moving in a circle with constant speed 0.1c measured by an observer fixed at the center of the circle. **5**

2. (a) Attempt any one.
   (i) Considering the Law of Conservation of momentum to be true in all inertial frames, derive the relation between the moving mass and rest mass. **10**
   (ii) Derive the relativistic kinetic energy equation **10**
   $K = m_0c^2\left[\frac{1}{\sqrt{1-u^2/c^2}} - 1\right]$. Hence show that it reduces to the Newtonian expression for kinetic energy in the non relativistic limit.

   (b) Attempt any one.
   (i) What is the relativistic mass & momentum of a photon of frequency ν ? Show that rest mass of a photon is zero. **5**
   (ii) The momentum of an electron observed in a magnetic deflection experiment is $3m_0c$; where m_0 is the rest mass of the electron and c is the velocity of light. Find the velocity of the electron. **5**

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3. (a) Attempt any one. 10

(i) Derive the transformation equations for electric field intensity $\vec{E}$.

(ii) Obtain expressions for the force between two identical electric point charges moving with equal velocity in an inertial frame of reference S. Discuss the significance of the result. 10

(b) Attempt any one. 5

(i) Show that the quantity $j_x^2 + j_y^2 + j_z^2 - \rho^2 c^2$ remains invariant under Lorentz transformations of current density $\vec{j}$ & volume density of charge ρ. 5

(ii) Show that a current carrying wire at rest in an inertial frame S is charged in frame S' moving with uniform velocity with respect to S.

4. (a) Attempt any one. 10

(i) Explain the relative nature of simultaneity using Minkowski space-time diagram. 10

(ii) Describe the large scale structure of the universe.

(b) Attempt any one. 5

(i) Write a short note on X-ray astronomy. 5

(ii) Explain in brief the Twin-Paradox.

5. (a) Attempt any one. 4

(i) What is the proper time interval between the occurrence of two events if in some inertial frame they are :- (p) Separated by 10^9 m and occur 5 seconds apart; (q) Separated by 7.5×10^8 m and occur 2.5 seconds apart; (r) Separated by 5×10^8 m and occur 1.5 seconds apart ? Assume $c = 3 \times 10^8$ m/s.

(ii) Assuming the Lorentz transformation equations, show that if two events are simultaneous in one inertial frame, they may not be in another. 4

(b) Attempt any one.

(i) The density of gold is 19.3 gm / cm^3 when it is at rest relative to an observer. What is its density when it is moving with relative velocity 0.9c ? 4

(ii) Calculate the velocity of a body when its total energy is three times its rest energy. 4

(c) Attempt any one.

(i) Show that $\vec{E} \cdot \vec{B}$ is invariant under Lorentz transformations. 4

(ii) If a field is purely electric in inertial frame S, describe it in another inertial frame S'. 4

(d) Attempt any one.

(i) What is gravitational redshift ? 3

(ii) A star is approaching the earth at a speed of 0.02 c. Calculate the wavelength shift for the wavelength 5000 $\overset{0}{A}$ emitted by the star. 3