Time: 2.5 Hours **Total Marks: 75**

N. B. 1) All questions are compulsory.
2) Figures to the right indicate marks.

Q. 1 A) Attempt any one from the following. **(8)**

i) Define the double integral of a bounded function $f : S \to \mathbb{R}$ where $S = [a, b] \times [c, d]$ is a rectangle in $\mathbb{R}^2$. Further show with usual notations $m(b-a)(d-c) \le \iint_S f \le M(b-a)(d-c)$.

ii) Let U be an open set in $\mathbb{R}^2$ containing the rectangle [a, b] × [c, d]. Suppose f: $U \to \mathbb{R}$ is continuously differential function. Show that $g'(x) = \int_c^d \frac{\partial f(x,y)}{\partial x} dy$ where $g(x) = \int_c^d f(x,y)dy \; \forall x \in [a, b]$.

B) Attempt any two from the following. **(12)**

i) Prove that a continuous function is integrable for a rectangular domain in $\mathbb{R}^2$.

ii) Evaluate $\iiint_S 4\, dv\, , where\ S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \le z \le 1\}$

iii) Evaluate $\iint_S (x^2 + y^2)\, dxdy$ where S is the region in the XY-plane bounded by the curves $x^2 - y^2 = 1,\ x^2 - y^2 = 2, xy = 2, xy = 4$.

Q.2 A) Attempt any one from the following. **(8)**

i) Let U be an open set in $\mathbb{R}^n$ and $\alpha : [a, b] \to U$ be a parameterization of curve Γ. If $f, g : U \to \mathbb{R}$ are continuous function, then prove that

$$\int_\Gamma (cf + dg) = c\int_\Gamma f + d\int_\Gamma g$$

where c, d are real constants. Further show that $\int_\Gamma f = \int_{\Gamma_1} f + \int_{\Gamma_2} f$, where Γ_1 and Γ_2 are restrictions of α to $[a, c]$ and $[c, b]$ where $a < c < b$.

ii) State and prove the Green's theorem for a rectangle.

B) Attempt any two from the following. **(12)**

i) Evaluate the line integral of scalar field $f(x, y, z) = x + y + z$ along the curve C with parameterization $\gamma(t) = (\sin t, \cos t, t),\ 0 \le t \le 2\pi$.

ii) Use the Greens' theorem to evaluate $\oint_C (y + 3x)dx + (2y - x)dy$ where C is the ellipse $x^2 + \frac{y^2}{4} = 1$.

iii) Evaluate the line integral $\int_C\ x^2dx + xy\, dy + dz,$ where C is parametrized by $c(t) = (t,\ t^2, 1), 0 \le t \le 1$.

Q.3 A) Attempt any one from the following. **(8)**

i) State Divergence Theorem for a solid in 3-space (or $\mathbb{R}^3$) bounded by an orientable closed surface with positive orientation and prove the Divergence Theorem for cubical region.

ii) For the surface $\bar{r}(u,v)$ described by the vector equation $\bar{r}(u,v) = X(u,v)\hat{\imath} + \; Y(u,v)\hat{\jmath} + Z(u,v)\hat{k}, (u,v) \in T$ where X, Y, Z are differentiable on T, define the fundamental vector product $\frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial u}$. If C is a smooth curve lying on the surface, $C = \bar{r}(\propto(t)),\ \propto; [a,b] \to T$, then show that $\frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial u}$ is normal to C at each point.

B) Attempt any two from the following. **(12)**

i) Let $S = \bar{r}(T)$ be a smooth parametric surface in uv plane. Define area of S. If S is represented by an equation $z = f(x,y)$ then show that area of S is given by

$$\iint_T \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\ dxdy.$$

ii) Evaluate surface integral of scalar field f over the surface S where $f(x,y,z) = xyz$, S is the surface of the cone $z^2 = x^2 + y^2$ between $z = 1$ and $z = 2$.

iii) Verify the Gauss Divergence Theorem for $F(x,y,z) = (yz, zx, xy)$ over the surface of the sphere $x^2 + y^2 + z^2 = 9$.

Q. 4 Attempt any three from the following. **(15)**

i) Reverse the order of integration and evaluate $\int_0^1 \int_{\sqrt{x}}^1 \sqrt{1+y^3}\, dydx$.

ii) State Fubini's theorem and hence evaluate $\iint_D f\, dA$ where $f(x,y) = 5xy$ and D is the region bounded by $x = \pm 1$ and $y = \pm 2$.

iii) Evaluate the following integral by showing it is independent on the path.

$$\int_{(1,0,0)}^{(0,1,1)} \sin y \cos x\, dx + \cos y \sin x\, dy + dz\,.$$

iv) Using Green's Theorem, find the area of the region D where D is the asteroid $r(t) = (\cos^3 t)\bar{\imath} + (\sin^3 t)\bar{\jmath},\ 0 \le t \le 2\pi$.

v) Evaluate surface area of the surface S cut from the plane $x + y + z = 5$ by the cylinder whose walls are $x = y^2$ and $x = 2 - y^2$.

vi) Using the Stoke's Theorem, evaluate the surface integral $\iint_S (curl\ F)\cdot \bar{n} dS$ where $F(x,y,z) = (x - z, x^3 + yz, -3xy^2)$, S is the surface of the cone $z = 2 - \sqrt{x^2 + y^2}$ above the plane $z = 0$.

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