

QP Code : 12772

(2½ Hours)

[Total Marks : 75

- N.B. :** (1) All questions are compulsory with internal options.
 (2) Figures to the right indicate full marks.
 (3) Symbols have the usual meaning unless stated otherwise.
 (4) Use of non-programmable calculators and log tables is allowed.

1. (a) Attempt **any one** of the following:

- i) Show that for an electrostatic field, $\text{div } \vec{E} = \rho / \epsilon_0$ and $\text{curl } \vec{E} = 0$.

Examine whether the following is a possible electrostatic field:

$$\vec{E} = k[xy\hat{x} + 2yz\hat{y} + 3zx\hat{z}] \text{ here } k \text{ is a constant with appropriate units.}$$

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- ii) Use Gauss' Law to find the electrostatic field outside and inside a uniformly charged sphere of charge density ρ and radius R . Hence calculate the electric potential in both the regions, taking the reference point at infinity.

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(b) Attempt **any one** of the following:

- i) An infinite plane carries a uniform surface charge density σ . Find \vec{E} . Hence find the \vec{E} for points inside and outside a charged parallel plate capacitor, making the appropriate assumptions.

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- ii) Find the electric field at a distance 's' from an infinitely long, straight wire carrying a uniform linear charge density ' λ '. Hence calculate the electric potential assuming $P(s_0, \theta_0, z_0)$ as the reference point.

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2. (a) Attempt **any one** of the following:

- i) Obtain an expression for potential due to bound charges, for a polarized dielectric in terms of σ_b and ρ_b .

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- ii) Using Biot-Savart's law, show that $\vec{\nabla} \cdot \vec{B} = 0$. Also explain the physical significance of the result.

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(b) Attempt **any one** of the following.

- i) Obtain Gauss's law for a polarized dielectric in both differential and integral form.

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- ii) For a very long solenoid consisting of 'n' closely wound turns per unit length on a cylinder of radius 'R' and carrying a steady current

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T, assume that $B_s = 0$ and $B_\phi = 0$. Calculate \vec{B} both inside and outside the solenoid.

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3. (a) Attempt **any one** of the following:

(i) Explain why it was necessary to modify Ampere's law in its original form. Explain how Maxwell modified it.

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(ii) Obtain the expression for energy stored in a magnetic field. What is the energy density?

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(b) Attempt **any one** of the following:

(i) Show that in a linear medium $\vec{B} = \mu \vec{H} = \mu_0 (1 + \chi_m) \vec{H}$.

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(ii) Give the physical interpretation of surface current density \vec{K}_b .

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4 (a) Attempt **any one** of the following:

(i) Derive the electromagnetic wave equation in vacuum, where there are no free charges or currents. For a plane wave, show that the electric field, magnetic field and the direction of propagation are mutually perpendicular.

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(ii) State and prove Poynting's theorem and obtain its differential version.

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(b) Attempt **any one** of the following:

(i) For a certain medium $\epsilon = 17.7 \times 10^{-12} \text{ C}^2/\text{Nm}^2$ and $\mu = 4\pi \times 10^{-7} \text{ N/A}^2$. Find the velocity of plane electromagnetic wave in the medium. Also find the refractive index of the medium. ($c = 3 \times 10^8 \text{ m/s}$)

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(ii) Show that in case of plane monochromatic waves the contributions from electric and magnetic fields towards the electromagnetic energy density are equal.

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5. (a) Attempt **any one** of the following:

(i) Consider the xy plane to be a grounded, conducting plane. Charges q_1 and q_2 are held fixed at (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively where both z_1 and $z_2 > 0$. Locate the image charges and hence find $V(x, y, z)$ for points above the plane.

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(ii) A hollow spherical shell carries charge density $\rho = k/r^2$ for $a \leq r \leq b$. Find \vec{E} in the regions $r < a$ & $a < r < b$

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