Phy -I
9/10
Maths

QP Code : 12820

(2½ Hours) [ Total Marks : 75

N.B. : (1) All questions are compulsory.
(2) Figures to the right indicate full marks.
(3) Symbols have the usual meaning unless stated otherwise.
(4) Use of log tables and non programmable calculators is allowed.

1. (A) Attempt any one:-
(i) Solve the following equations.
a) $(e^x+1)\cos x\,dx + e^x \sin x\,dy = 0$ 5
b) $[y(1+\frac{1}{x}) + \cos y]\,dx + (x + \log x - x\sin y)\,dy = 0$ 5
ii) Solve the following equation. 10

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad \text{if } u(x,o) = \frac{1}{2}x(\ell - x)$$

by using method of separation of variables.

(B) Attempt any one:-
i) Uranium disintegrates at a rate proportional to the amount present at any instant. If $m_1$ and $m_2$ grams of uranium are present at time $t_1$ and $t_2$ respectively. Show that half life of uranium is 5

$$\frac{(t_1 - t_2)\log_e 2}{\log_e \frac{m_1}{m_2}}$$

ii) Solve the given equation by sucessive integration.

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^x$$ 5

2. (A) Attempt any one:-
i) Write the Fourier expansion of f(x). Obtain the complex form of Fourier series expansion. Consider a single pulse of height 'A' and with '2a' defined by function $f(t) = A \quad -a \le t \le a$
$= 0$ other wise.
Obtain complex form of Fourier Integral. 10

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CM-Con. : 2268-15.

ii) What is Fourier transform? Obtain the Fourier transform of 10

$f(x) = 1 \quad |x| \leq a$

$= 0 \quad |x| \geq a$

(B) Attempt any one :

i) Expand the function $f(x) = x \quad [-\pi \leq x \leq \pi]$ as a Fourier series. 5

ii) Find cosine transform of $f(x) = e^{-x}$ 5

3. (A) Attempt any one :

i) Prove the fundamental law of thermodynamics for infinitesimal general interaction : $TdS = dU + PdV$ 10

ii) What do you mean by thermal interaction and external parameters? Prove that in case of thermal interaction $\beta(U) = \beta'(U')$ at equilibrium. 10

(B) Attempt any one :

i) Show that the entropy of a system in a heat bath is given by 5

$$S = -K \sum_r P_r \ell n P_r$$

ii) Define particle states and system states. Explain with one example each. 5

4. (A) Attempt any one:

i) Derive Maxwell-Boltzmann distribution law. Evaluate $e^{\alpha}$. 10

ii) What are bosons? Derive Bose-Einstein distribution law. 10

(B) Attempt any one:

i) Consider a box with its top surface divided in two cells of areas in the ratio 1:2 and 12 idential balls are thrown in the box in a completely random manner. Calculate the probability of the distribution of equal number of balls in each box and probability of the most probable distribution. 5

ii) Calculate the average energy of a Planck's oscillator of frequency $1.5 \times 10^{14}$ Hz at T = 1800 K

Given $h = 6.63 \times 10^{-34}$ Js,

Boltzmann constant (k) = $1.38 \times 10^{-23}$ J/K. 5

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5. (A) Attempt any one:

i) Solve

$$\cos(x+y)\,dy = dx$$ 4

ii) Find the value of $\lambda$ for which the differential equation $(xy^2 + \lambda x^2 y)\,dx + (x+y)\,x^2 dy = 0$ is exact. Solve the equation for this value of $\lambda$. 4

(B) Attempt any one:

i) $f(x) = x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$, show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$ 4

ii) Show the $F_c'(k) = k F_s(k) - \sqrt{\frac{2}{\pi}} F(0)$ where signs have their usual meaning. 4

(C) Attempt any one:

i) A system of 5 distinguishable particles has total energy 12 units. There are four non-degenerate energy levels with energies 1,2,3 & 4 units accessible to all particles. Find the various distributions possible for the system and the number of accessible microstates far each distribution. Which is the most probable distribution? 4

ii) In a random distribution of 10 particles between two boxes with equal probability, calculate (a) probability of distribution (3,7), (b) total no. of macrostates and microstaes, (c) No. of microstates in macrostate (3,7) and (d) most probable macrostate. 4

(D) Attempt any one:

i) With the help of neat diagram, explain the experiment which shows that a black body at a given temperature radiates energy faster than any other body. 3

ii) State Planck's radiation formula for black body radiation. Hence obtain Wien's displacement law. 3