VCD/310323 SYBSC- SEM IV - MATHEMATICS II- 75MARKS- 2½HRS

NOTE : 1) All questions are compulsory.

2) For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any two subquestions (each 6marks) from part (b).

3) For Q.4 , attempt any three. (each 5 marks)

Q.1. (a) Attempt any one. [each 8Mks]

1) Define an invertible linear transformation and Prove that Inverse of a linear transformation if exists is unique.

2)Let V,W be a finite dimensional real vector Spaces and $T:V \rightarrow W$ be a linear transformation then prove that Rank of T=Rank of m(T)

(b) Attempt any two. [each 6Mks]

1) Verify Rank-Nullity theorem for linear transformation $T:\mathbb{R}^4 \rightarrow \mathbb{R}^2$ as $T(x,y,z,w)=(x+y,z+w)$

2)Show that the following vector Spaces are isomorphic by explicitly defining an isomorphism $\mathbb{R}^3$ and $P_2[x]$ where $P_2[x]$ is set of all polynomial of degree≤ 2

3)For a real vector Spaces V,W, linear transformation $T:V \rightarrow W$prove that i)$T(-x)=-T(x)\ \forall x \in V$ ii)$T(x-y)= T(x)-T(y)$for $x,y \in V$

iii)$T(\alpha x+\beta y)=\alpha T(x)+\beta T(y), \forall \alpha,\beta \in \mathbb{R}, x,y \in V$

Q.2. (a) Attempt any one. [each 8Mks]

1)State and prove Cauchy-Schwarz Inequality for an inner product space (V,<>)

2) Prove that the a parallelogram is a rhombus iff diagonals are perpendicular to each other .

(b) Attempt any two. [each 6Mks]

1)In an inner product space $V=\mathbb{R}^2$ with $\langle (x_1,x_2),(y_1,y_2)\rangle=4x_1y_1+9x_2y_2$

Find i) $\|(1,-1)\|$ ii) distance between(4,2),(-1,3) in this space iii) prove that the vectors (2,1),(-9,8) are orthogonal vectors in this space

2)Find the orthonormal basis corresponding to the basis of $\mathbb{R}^3$ {(1,0,3),(2,1,1)} using Gram Schmidt orthogonalisation process

3)Let (V,<>)be an inner product space &let $d(x,y)=\|x-y\|$ for $x,y\in V$ then prove i)$d(x,y)\geq 0$ for $x,y\in V$. ii)$d(x,y)=d(y,x)\ \forall\ x,y\in V$ iii)$d(x,z)\leq d(x,y)+d(y,z)\ \forall\ x,y,z\in V$

Q.3. (a) Attempt any one. [each 8Mks]

1)i) Let A be n×n real matrix.Let $\lambda\in R$ be an eigen value of A.Let $W=\{x=(x_1,x_2,...,x_n)\in R^n/Ax=\lambda x\}$ be the set of all eigen vectors of A associated with λ then prove that W is a vector subspace of R^n.

ii)Prove that eigen values of n×n real diagonal matrix A are the diagonal entries of A.

2)Let A be 2×2 matrix and f(x) be the characteristics polynomial of A then prove that f(A)=0.

(b) Attempt any two. [each 6 Mks]

1)Prove that similar matrices have same eigen values.

2) Find eigen value and corresponding eigen vector of

$$A=\begin{bmatrix}3 & 2 & -1\\ 0 & 1 & 1\\ 0 & 0 & -2\end{bmatrix}$$

3)Suppose eigen values of a 3×3matrix A are 1,2,3 then prove that A^{-1} exists and also find $\det(A^{-1})^T$

Q.4.Attempt any three. [each 5 Mks]

1)Find the matrix associated with the linear transformation $T:R^3\rightarrow R^4$ with respect to the standard bases of R^3&R^4 where $T(x,y,z)=(4x+9y,3y-7z,x+y+z,7x+6y+2z)$

2) Prove that $T:R^3\rightarrow R^4$ as $T(x,y,z)=(9x+7y,x-y+6z,4y-3z,6z+y)$ is a linear transformation.

3) Find projection of $p(x)=x$ on $q(x)=x^2+1$ using $<p,q>=\int_0^1 p(x)q(x)dx$ in C[0,1]

4)Find orthogonal complement of the subspace $W=\{(x,y,z)\in R^3/z=0\}$ in R^3.

5) Find eigen value of linear transformation $T:R^3\rightarrow R^3$ given by $T(x,y,z)=(x+z,-y+3z,2x+2z)$

6)Find the quadratic form associated with the symmetric matrix

$$A=\begin{bmatrix}4 & 2\\ 2 & -1\end{bmatrix}$$

XXXXXXXX