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2)Let $S\neq\varphi$, open subset of R^2,$(a,b)\in S$ be stationary point of f. Suppose f(x,y) possesses continuous second order partial derivative in some neighborhood of(a,b).Let$A=f_{xx}(a,b)$,$B=f_{xy}(a,b)$,$C=f_{yy}(a,b)$. Let $\Delta=AC-B^2$.prove that i) if $A>0$, $\Delta>0$ then f has local minimum at (a, b) ii) if $A<0$, $\Delta>0$ then f has local maximum at (a, b)

(b) **Attempt any two. [each 6 Mks]**

1)Find the total derivative of f at (1,1,1) in Jacobian form and also in linear transformation form where $f:R^3\to R^3$ as f(x, y, z) =(x+y, y+z, z+x)

2)Define the Differentiability of a vector valued function $f:R^n\to R^m$ at $a\in R^n$.Let $f:R^n\to R^m$ be differentiable at $a\in R^n$ then prove that αf is differentiable at a,$\alpha\in$ R and $D(\alpha f)(a)=\alpha Df(a)$

3)Find the maximum possible rate of change $f(x, y, z) =I_n(x+y+z)$ at (1, 2,3).also find the direction in which such a maximum rate of change occurs.

Q.4.Attempt any three. [each 5 Mks]

1)Find f_x, f_y at (0, 0) if exists for $f:R^2\to R$ is defined as $f(x, y) = x^3/(x^2+y^2)$ if$(x, y)\neq(0, 0)$

=0 if (x, y) =(0, 0)

2) For following function f, find the real$\theta\in(0, 1)$ if exists satisfying $f(b)-f(a) =\nabla f(a+(b-a)\theta).(b-a)$ where $f(x, y) =x^2+x+y$, a=(0,0) , b=(1,-1)

3) Using chain rule find the total derivative of $f(x, y, z) =xy^2+yz^2+zx^2$, $x(t) =e^t$, y(t)=sin t, z(t)=cos t.

4) Find directional derivative of f at a in direction of u $f(x, y, z) =z^2-x^2-y^2$, a=(1, 0,1), u=(4, 3,0)

5) Let $f:R^2\to R^3$, $g:R^3\to R^2$ as $f(x, y) =(3x^3+4xy,y^2+3x,x^2+y^2) =(f_1, f_2, f_3)$, $g(u, v, w) =(uvw, u^2+v^2+w^2) =(g_1, g_2)$ Find J(f(x, y)), J(f(g(a)), a=(1, -1, -1)

6) Locate all critical points of $f(x, y) =x^3-6xy+3y^2-2yx+4$

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VCD/ SYBSC- SEM IV - MATHEMATICS I- 75MARKS- 2½HRS

NOTE : 1)For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any two subquestions (each 6marks) from part (b).For Q.4 , attempt any three. (each 5 marks)

Q.1. (a) Attempt any one. [each 8Mks]

1)Define a continuity of a vector valued function $f:S \to R^m, S \neq \varphi$ subset of R^n.Prove that for nonempty subset S of R^n, $f,g:S \to R^m$ continuous at $a \epsilon S$ then f-g is continuous at $a \in S$.

2) Prove that sequence $w_n=(s_n,t_n)$ in R^2 converges to a limit $w=(s,t) \in R^2$ iff $(S_n) \to s$ &$(t_n) \to t$

(b) Attempt any two. [each 6Mks]

1)Define norm of x where $x=(x_1,x_2,\ldots,x_n) \in R^n$ and prove that $||x+y|| \leq ||x||+||y||$ $x,y \in R^n$

2)Show that $S=\{(x,y) \in R^2/3x+4y<12\}$ is an open set.

3)Define $f:R^2 \to R$ defined by $f(x,y)=x\sin\frac{1}{y}+y\cos\frac{1}{x}$ for $(x,y) \neq (0,0)$

=0. otherwise Find $\lim_{(x,y) \to (0,0)} f(x, y)$ if exists

Q.2. (a) Attempt any one. [each 8Mks]

1)State and prove Euler's theorem for function of two variable.

2)Define a Differentiability of a scalar valued function $f:S \to R, S \neq \varphi$ subset of R^n at point $a \in S$. Prove that for nonempty subset S of R^n, $f:S \to R$ be differentiable at $a \in S$ then $\partial f(a)/\partial x_i$ exists for i=1, 2,..n

(b) Attempt any two. [each 6Mks]

1)Evaluate total derivative of f using definition at the mentioned point
$f(x, y, z) = x^2+2y^2+3z$, $a=(1,-1,0)$

2)Let $f:R^2 \to R$ as $f(x, y) = 3\sin x + y\cos x$. Find $f_x, f_y, f_{xx}, f_{yx}, f_{xy}, f_{xxy}$.

3)Let $f:R^2 \to R$ be non constant differentiable function, $k \in R$, $f(x, y) = k$ describes the curve C having tangent at each of ots points then prove that i)gradient vector ∇f is normal to C ii) the directional derivative of f is zero along C

Q.3. (a) Attempt any one. [each 8Mks]

1)Let $S \neq \varphi$, open subset of R^n , $a \in S$& $f:S \to R$ be a scalar field. Let f be differentiable at a. If f has a local maximum or local minimum at a then prove that $\nabla f(a) = 0$.