

- N.B.: (1) All questions are compulsory.  
 (2) Figures to the right indicate full marks.  
 (3) Use of calculator is allowed.

Q.1 Attempt all sub questions:

(20)

(A) State True or False and correct if necessary:

(10)

- (i) If a random variable follows Uniform distribution over  $(-1, 1)$ , then its mean is 0.  
 (ii) If a r.v. follows Exponential distribution with parameter 3, its mean is  $1/3$ .  
 (iii) If  $X \sim N(5, 9)$  then 5<sup>th</sup> central moment is equal to 5.  
 (iv) Square of Standard Normal Variable has Chi-Square distribution with n degrees of freedom.  
 (v) If  $F(x) = 1 - F(-x)$ , then the distribution is symmetric.

(B) Answer the following.

(10)

- (i) If a r.v. follows Uniform distribution over  $(-5, 5)$ , write down its probability density function(p.d.f.).  
 (ii) Identify the distribution and find its mean if p.d.f. is given by  

$$f(x) = 60x^3(1-x)^2, \quad 0 < x < 1$$

$$= 0 \quad \text{otherwise}$$
  
 (iii) If r.v.  $X \sim N(18, 25)$ , what is the third quartile  $Q_3$  of the distribution.  
 (iv) When do you apply Yate's Correction? Why?  
 (v) What is the Confidence Interval for the ratio of variance based of F distribution.

Q.2 Attempt any TWO questions:

(20)

- (a) If  $X$  follows  $\text{Gamma}(\alpha, \lambda)$ , write its p.d.f. and obtain mean and variance. (10)  
 (b) If a random variable  $X$  follows Beta distribution of type I then state its p.d.f. and Obtain rth order raw moment, mean and variance. (10)  
 (c) State and prove forgetfulness property of Exponential distribution. Suppose the length of shower (in minutes) has Exponential distribution with parameter  $\theta=3$ . If the shower is already lasted for 3 minutes, what is the probability that it will last for at least two more minutes? (10)

Q.3 Attempt any TWO questions:

(20)

- (a) i. If  $X \sim N(\mu, \sigma^2)$  then derive expression for even ordered central moments. (8)  
 ii. State the Central Limit Theorem. (2)  
 (b) i. State five properties of Normal distribution. (5)  
 ii. Derive the mode of  $N(\mu, \sigma^2)$  distribution. (5)  
 (c) A test of solved examples was given to 600 candidates. The marks obtained in the test were assumed to follow normal distribution with mean 15 and s.d. 16. Find i) how many candidates obtained marks between 13 and 16. ii) how many scored below 12? iii) how many scored above 18? iv) what is the probability that a candidate chosen at random had scored between 14 and 18? (10)

Q.4 Attempt any TWO questions:

(20)

- (a) i. Derive the distribution of reciprocal of F variate. (5)  
 ii. State the properties of Chi-Square distribution with n degrees of freedom. (5)  
 (b) i. Prove that odd ordered moments of t-distribution are equal to zero. (5)  
 ii. State the application of Chi-Square distribution. (5)  
 (c) i. Two types of batteries are tested for their length of life and the following data are obtained: (5)

	No. of sample	Mean life(in hrs)	Variance
Type A	9	600	121
Type B	8	604	144

Use 5% level of significance, should we conclude that they have same length of life.



ii. A random sample of size 10 from a normal population gave the following value.

$$\sum_{i=1}^{10} (x_i - \bar{x})^2 = 235. \quad (5)$$

Test the hypothesis that population variance is 32. Take 1% level of significance.

Q.5 Attempt any TWO questions: (20)

(a) i. If r.v.  $X \sim U(a, b)$  with mean=5, variance=3, obtain a and b. Also find  $P(X < 4)$ . (5)

ii. The lifetime of certain device assumed to follow Exponential distribution with

Mean 300 hours. What is the probability that such device will last

(1) between 150 and 300 hours (2) at least 180 hours. (5)

(b) Obtain mean and variance of normal distribution with parameters  $N(\mu, \sigma^2)$  (10)

(c) i. State the properties of t- distribution. (5)

ii. State the properties of F- distribution. (5)

XXXXX

munotes.in