SYBSC Maths IV

07/03/15 Date 7/3/15

VCD -- - MATHS II- SYBSC - SEM IV EXAM - 75 MARKS - 2.5HRS -160

Note:: 1) All questions are compulsory.

2)For Q.1, Q.2, Q.3, attempt any one subquestion (each 8 mks)from part (a), and any three subquestions (each 4 mks) from part(b)

3) For Q.4 Attempt any three.(each 5 mks)

Q.1 (a) Attempt any one [Each 8]

1) Verify Rank-Nulity Theorem for $T:\mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x,y,z) = (x+y,z)$

2) Let V, U be vector spaces over $\mathbb{R}$. If $T: V \to U$ is a linear transformation then prove that i) $T(0) = 0$ ii) $T(-x) = -T(x)$ $\forall x \in V$

Also Define a Linear Transformation and Check whether following is a linear transformation or not.

$T: \mathbb{R}^2 \to \mathbb{R}$ defined by $T(x,y) = 2x + y$

Q.1 (b) Attempt any three. [Each 4]

1) Let V, U be a vector spaces over $\mathbb{R}$ and $T: U \to V$ be a linear transformation then define Image of T (Img T) and prove that Img T is a subspace of V.

2) Define a matrix associated with linear transformation and find matrix of linear transformation for following linear transformation

$T: \mathbb{R}^2 \to \mathbb{R}^3$ as $T(x,y) = (x+y, 2y, x-y)$ with respect to natural basis

$\{(1,0,0), (0,1,0), (0,0,1)\}$ of $\mathbb{R}^3$ and $\{(1,0), (0,1)\}$ of $\mathbb{R}^2$

3) Let V, V' be m-dimensional and n-dimensional vector spaces over $\mathbb{R}$ and

$B = \{e_1, e_2, \dots, e_m\}$ & $B' = \{e_1', e_2', \dots\dots, e_n'\}$ be ordered bases of V, V' respectively.

Prove that if $T_1: V \to V'$ $T_2: V \to V'$ are linear transformation Then

$$m(T_1 + T_2) = m(T_1) + m(T_2)$$

4) Define row rank, column rank and rank of a matrix A and find rank of

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix}$$

Q.2 (a) Attempt any one [Each 8]

1) Define the determinant of a matrix of order n and derive the formula for determinant of

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

2) Prove that if A is matrix of order n then det A = det (A^T) where A^T = transpose of A.

Q.2 (b) Attempt any three. [Each 4]

1) Define Determinant of a matrix using Laplace Expansion and use Laplace Expansion by 2nd row to find determinat of following matrix

$$A = \begin{bmatrix} 3 & 2 & 5 \\ 0 & 3 & 2 \\ 2 & 1 & 1 \end{bmatrix}$$

2) Define Vandermonde determinant and solve

$$A = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \\ 4 & 9 & 16 & 25 \\ 8 & 27 & 64 & 125 \end{vmatrix}$$

3) Define linearly independant and linearly dependant vectors in $\mathbb{R}^n$.

Check whether following vectors are linearly independant or linearly dependant

$$< 1,0,0 >, < 0,0,1 >, < 0,1,0 >$$

4) Find inverse of a matrix using adjoint method

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Q.3 (a) Attempt any one [Each 8]

1) Find eigen values and eigen vectors

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{bmatrix}$$

2) Verify Cayley Hamilton Theorem and hence find A^{-1} if exist.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

Q.3 (b) Attempt any three. [Each 4]

1) Prove that Zero is eigen value of a matrix iff matrix is singular.

2) Define an eigen value of a matrix and prove that the eigen values of a matrix and its transpose are same.

3) Define similar matrix and prove that if A & P are $n \times n$ matrices and P is nonsingular then A &$P^{-1}AP$ have same eigen values.

4) Prove that the eigen values of a diagonal matrix are diagonal elements.

Q.4 (a) Attempt any three [Each 5]

1) Let V be a finite dimensional vector space over $\mathbb{R}$ and $T: V \to V$ be a linear transformation Then prove that T is invertible iff T is one-one.

2) Let U, V, W be vector spaces over $\mathbb{R}$ and $T: U \to V, S: V \to W$ be a linear transformation Then prove that the composition map $S \circ T: U \to W$ is also a linear transformation.

3) Find volume of parallelepiped bounded by three vectors V_1, V_2, V_3

$$V_1 =< 2,3,5 >, V_2 =< -2,1,-6 >, V_3 =< -1,7,-1 >$$

4) Find area of parallelogram formed by edges V_1, V_2 where V_1, V_2

$$V_1 = 2i - 3j, V_2 = 3i + 8j$$

5) Prove that eigen vectors corresponding to distinct eigen values are linearly independent.

6) Define the following term

i) Eigen value of a matrix ii) Eigen vector of a matrix

iii) Diagonalisable matrix iv) Similar matrices

v) Characteristic polynomial of a matrix