SYBsc Maths-I SEM IV
5/3/15

05/03/15 (175)

VCD -- - MATHS I- SYBSC - SEM IV EXAM - 75 MARKS - 2.5HRS -160

Note:: 1) All questions are compulsory.

2)For Q.1, Q.2, Q.3, attempt any one subquestion (each 8 mks)from part (a), and any three subquestions (each 4 mks) from part(b)

3) For Q.4 Attempt any three.(each 5 mks)

Q.1 (a) Attempt any one [Each 8]

1) Solve nonhomogeneous differential equation $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$

2) Define an exact differential equation and solve following

$(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$

Q. 1 (b) Attempt any three. [Each 4]

1) Define an order and degree of differential equation and Write order and degree of each of following differential equation

i) $\frac{dy}{dx} + kx = 0$ ii) $k(y'')^2 = [1 + (y'')^2]^3$ iii) $\left(\frac{d^2y}{dx^2}\right) + 2\frac{dy}{dx} + y = 0$

2) Solve $\left(y - x\frac{dy}{dx}\right) = m(y^2 + \frac{dy}{dx})$ using separation of variable.

3) Prove that The Bernoulli Differential equation $\frac{dy}{dx} + Py = Qy^n$ reduces to linear differential equation by transformation $z = y^{1-n}$

4) Define linear differential equation and solve $y\log y\, dx + (x - \log y)dy = 0$

Q.2 (a) Attempt any one [Each 8]

1) Define double integral and write properties of double integral and evaluate

$$\int_0^1 \int_0^{\sqrt{2}} (x^2 + y^2)dy\, dx$$

2) Write the note on application of double integral to find area of closed bounded region R and find area of the region R bounded by $y = 2x$, $y = 4$ and y-axis .

Q. 2 (b) Attempt any three. [Each 4]

1) Evaluate the triple integral

$\iint_E x^2 e^y dv$ where E is bounded by the parabolic cylinder $z = 1 - y^2$ and the planes

$z = 0, x = 1, x = -1$

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2) Find mass and center of massof a triangular lamina with vertices (0,0),(1,0),(0,2) if the density function is $\rho(x, y) = 1 + 3x + y$

3) Find the average value of $F(x, y, z) = \frac{1}{\sqrt{1-x^2-y^2-z^2}}$ over the sphere

$$x^2 + y^2 + z^2 = 1$$

4) Sketch the region and write an equivalent double integral with order of integration reversed $\int_0^1 \int_1^{e^x} dy\, dx$

Q.3 (a) Attempt any one [Each 8]

1) Define Potential function and Find Potential function for

$$F = (e^x \cos y + yz)i + (xz - e^x \sin y)j + (xy + z)k$$

2) Define circulation around the curve, flow along the curve and Find the circulation and flux of the field $F = -yi + xj$ around and across the closed semicircular path that consists of the semi circular arch

$r_1(t) = (a\, cost)i + (a\, sint)j, \quad 0 \le t \le \pi$followed by line segment

$$r_2(t) = t\, i \qquad -a \le t \le a$$

Q. 3 (b) Attempt any three. [Each 4]

1) Define the gradient field of a differentiable function f and find gradient of

$$f(x, y) = xyz$$

2) $F = (x - z)i + xk$is the velocity field of a fluid flowing through a region in space. Find the flow along the curve $r = (cost)i + (sint)k, \; 0 \le t \le \pi$

3) Define work done over a smooth curve by force F and find the work done by $F = 3x^2 i + (2xz - y)j - zk$ over the curve

$r(t) = t\, i + t^2\, j + t^3 k, \; 0 \le t \le 1$from (0,0,0) to (1,1,1)

4) Evaluate $\oint_C y^3 dx - x^3\, dy$ where C is the positively oriented circle of radius 2 centered at the origin using Green's theorem

Q.4 Attempt any three [Each 5]

1) Solve homogeneous differential equation $x(x - y)dy + y^2 dx = 0$

2) Using Rule 3 to find an integrating factor , Solve following

$$(2y + x^2y^3)dx + x(2 - 2x^2y^2)dy = 0$$

3) State Fubini's theorem and evaluate $\iint_R (x - 3y^2)dA$ where

$$R = \{x, y/0 \leq x \leq 2, 1 \leq y \leq 2\}$$

4) Use polar coordinate to find volume of given solid

Under the cone $z = \sqrt{x^2 + y^2}$ and above the disk $x^2 + y^2 \leq 4$

5) Evaluate $\int_c xy^4\, ds$ where C is the right half of the circle $x^2+y^2 = 16$.

6) Determine whether or not the vector field $F(x, y, z) = (z + y)i + zj + (y + x)k$ is conservative