[Additional Exam]

SYBSC Maths-I

08/05/2015

VCD -- - MATHS I- SYBSC - SEM IV EXAM - 75 MARKS - 2.5HRS -160

Note:: 1) All questions are compulsory.

2)For Q.1, Q.2, Q.3, attempt any one subquestion (each 8 mks)from part (a), and any three subquestions (each 4 mks) from part(b)

3) For Q.4 Attempt any three.(each 5 mks)

Q.1 (a) Attempt any one [Each 8]

1) Solve nonhomogeneous differential equation $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$

2) Define an exact differential equation and solve following

$(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$

Q. 1 (b) Attempt any three. [Each 4]

1) Define an order and degree of differential equation and Write order and degree of each of following differential equation

$$i)\ \frac{dy}{dx} + kx = 0 ii) k(y'')^2 = [1 + (y'')^2]^3 \quad iii) \left(\frac{d^2y}{dx^2}\right) - 2x\frac{dy}{dx} = 3y$$

2) Solve $(1 - x)dy - (1 + y)dx = 0$ using separation of variable.

3) Define linear differential equation and solve $\frac{dy}{dx} + \left(\frac{1-2x}{x^2}\right)y = 1$

4) Prove that The Bernoulli Differential equation $\frac{dy}{dx} + Py = Qy^n$ reduces to linear differential equation by transformation $z = y^{1-n}$

Q.2 (a) Attempt any one [Each 8]

1) Define double integral and write properties of double integral and evaluate

$$\int_0^1 \int_0^{\sqrt{2}} (x^2 + y^2)dy\, dx$$

2) Write the note on application of double integral to find area of closed bounded region R and find area of the region R bounded by $y = x$ & $y = x^2$ in the first quadrant

Q. 2 (b) Attempt any three. [Each 4]

1) Evaluate the triple integral

$$\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2)\, dz\, dy\, dx$$

2) Find mass and center of massof a triangular lamina with vertices (0,0),(1,0),(0,2) if the density function is $\rho(x, y) = 1 + 3x + y$

3) Find the average value of $F(x, y, z) = xyz$ over the cube bounded by the planes

$x = 2, y = 2, z = 2$in the first octant.

4) Sketch the region and write an equivalent double integral with order of integration reversed $\int_0^1 \int_y^{\sqrt{y}} dx\, dy$

Q.3 (a) Attempt any one [Each 8]

1) Define Potential function and Find Potential function for

$$F = (e^x \cos y + yz)i + (xz - e^x \sin y)j + (xy + z)k$$

2) Define circulation around the curve, flow along the curve and Find the circulation and flux of the field $F = xi + yj$ around and across the closed semicircular path that consists of the semi circular arch

$r_1(t) = (a\, cost)i + (a\, sint)j, \quad 0 \le t \le \pi$ followed by line segment

$$r_2(t) = ti \qquad -a \le t \le a$$

Q. 3 (b) Attempt any three. [Each 4]

1) Define the gradient field of a differentiable function f and find gradient of

$$f(x, y, z) = \sqrt{(x^2 + y^2 + z^2)}$$

2) Define work done over a smooth curve by force F and find the work done by $F = 3yi + 2xj + 4zk$ over the curve

$r(t) = t\,i + t\,j + tk, \quad 0 \le t \le 1$from (0,0,0) to (1,1,1)

3) $F = (x - z)i + xk$is the velocity field of a fluid flowing through a region in space. Find the flow along the curve $r = (cost)i + (sint)k, \quad 0 \le t \le 2\pi$

4) Let$F(x, y) = (x^2 - y)i + (y^2 - 2x)j$ find curl $F(x, y)$, div F.

Q.4 Attempt any three [Each 5]

1) Solve homogeneous differential equation $y^2 dx + x^2 dy = xydy$

2) Using Rule 2 to find an integrating factor , Solve following

$$(2xlogx - xy)dy + 2ydx = 0$$