\ ATKT 2015-16

Date : 01/10/15

VCD 011015 MATHS II- SYBSC - SEM III EXAM - 75 MARKS - 2.5HRS -80

Note:: 1) All questions are compulsory.

2)For Q.1, Q.2, Q.3, attempt any one subquestion (each 8 mks)from part (a), and any three subquestions (each 4 mks)from part(b)

3) For Q.4 Attempt any three.(each 5mks)

Q.1 (a) Attempt any one [Each 8]

1) Find the inverse of the following matrix and write it as product of elementary matrices

$$\begin{bmatrix} -4 & -2 \\ 5 & 5 \end{bmatrix}$$

2) State Rank Nulity theorem for a linear transformation $T: U \to V$ where U, V are vector spaces over $\mathbb{R}$ and verify it for following $T: \mathbb{R}^2 \to \mathbb{R}^2$ $\quad T(x, y) = (x + y, y)$

Q.1 (b) Attempt any three. [Each 4]

1) Let E be an elementary matrix obtained by some ERO on I and E' is also an elementary matrix obtained by undo (or inverse) ERO of the same on I. Then prove that E is invertible and $E^{-1} = E'$

2) Consider the following matrix and ERO. Show that $EA = A^*$ where A^* is the matrix obtained by performing the given ERO on A.

$$A = \begin{bmatrix} 0 & -1 \\ 2 & 4 \\ 1 & 3 \end{bmatrix} \text{ERO} = 3R_3$$

3) Define row rank, column rank and find row rank and column rank of following

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & -1 & -1 \end{bmatrix}$$

4) Define matrix associated with linear transformation and find matrix with respect to following linear transformation

$T: \mathbb{R}^3 \to \mathbb{R}^2$ as $T(x, y, z) = (x, y)$ with $B_1 = \{(1,0,0), (0,1,0), (1,1,1)\}$ basis of $\mathbb{R}^3$

$B_2 = \{(1,0), (0,1)\}$ basis of $\mathbb{R}^2$

Q.2 (a) Attempt any one [Each 8]

1) Prove that determinant of any triangular matrix is product of main diagonal elements

2) State and prove Cramer's rule

Q.2 (b) Attempt any three. [Each 4]

1) Find the inverse of following matrix using adjoint method

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

2) Find the area of the triangle whose vertices are (0,0), (3,4), (5,0)

3) Find volume of parallelepiped formed by three vectors V_1, V_2, V_3

$$V_1 = < 2,3,5 >, \quad V_2 = < -2,1,-6 >, \quad V_3 = < -1,7,-1 >$$

4) Let $A = (a_{ij})_{n\times n}$, B is matrix obtained by multiplying i^{th} row R_i by m then prove that
$\det(B) = m \det(A)$

Q.3 (a) Attempt any one [Each 8]

1) Let V be the set of continuous real valued function defined on $[-\pi, \pi]$

Define '+' and '.' on V as follows

$$(f+g)(t) = f(t) + g(t) \quad \forall\, t \in [-\pi, \pi]$$

$$(\alpha . f)(t) = \alpha . f(t) \quad \alpha \in \mathbb{R}, t \in [-\pi, \pi]$$

$$< f, g > = \int_{-\pi}^{\pi} f(t) g(t) dt$$

Show that $(V, <,>)$ is an inner product space with respect to + and .

2) Let $(V, <,>)$ be an inner product space then prove following

i) $\|u\| \geq 0$

ii) $\|u\| = 0$ iff $u = 0$

iii) $\|ku\| = |k| \; \|u\| \quad k \in \mathbb{R}$

iv) $\|u + v\| \leq \|u\| + \|v\|$

Q.3 (b) Attempt any three. [Each 4]

1) Verify Cauchy Schwarz inequality for the vectors with respect to Euclidean inner product

$$i)\ u = (-1,-1,0,2), \quad v = (2,-3,1,-2).$$

$$ii)\ u = (-2,-1,0,0), \quad v = (1,2,3,4)$$

2) Find $\|u\|, \|v\|, \quad d(u,v)$ for following in Euclidean space in $\mathbb{R}^3$

$$u = (15,-1,4), \quad v = (\pi,3,-1)$$

3) Define projection of a vector u along v, $v \neq 0$ and find projection of

$u = (1,-2,3)$ along $v = (2,-1,4)$

4) State and prove Pythagoras theorem. State generalized statement also.

Q.4 (a) Attempt any three [Each 5]

1) Define the row echelon form of a matrix and reduce following matrix in row echelon form.

$$A = \begin{bmatrix} 1 & 3 & 9 \\ 0 & 7 & 0 \\ 5 & 8 & 1 \end{bmatrix}$$

2) Solve the system of equation

$$x + 3y - 2z = 0$$
$$2x - y + 4z = 0$$
$$x - 11y + 14z = 0$$

3) Find the value of following determinant

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & -2 & -1 & 3 \\ 4 & 4 & 1 & 9 \\ 8 & -8 & -1 & 27 \end{bmatrix}$$

4) Check the following vectors in $\mathbb{R}^4$ are linearly dependent or independent.

$$< 4,0,0 >, < 6,3,0, >, < 2,3,6 >$$

5) Prove that an orthogonal set of nonzero vectors in an inner product space is linearly independent

6) Define an orthogonal complement of a set and prove that Let W_1, W_2 are subspaces of V then prove that if $(W_1 + W_2)^{\perp} = W_1^{\perp} + W_2^{\perp}$