VCD/ 25/10/23 **SYBSC- SEM III - MATHEMATICS I- 75MARKS- 2½HRS**

NOTE: 1) All questions are compulsory.

2) For Q.1, Q.2 and Q. 3 attempt any one sub-question (each 8 marks) from part (a), and any two sub-questions (each 6marks) from part (b).

3) For Q.4, attempt any three. (each 5 marks)

Q 1). (a) Attempt any one from following. **(08)**

1)Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, for $0 \le p \le 1$ is divergent series.

2) Prove that the series $\sum_{n=0}^{\infty} ar^n$ is convergent if and only if |r|<1.

b) Attempt any two from following. **(12)**

1)The telescoping series $\sum_{n=1}^{\infty}(t_n - t_{n-1})$ is convergent if and only if (t_n) is convergent sequence

2) Check convergence of the series $\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^2}$

3)Prove that Series $\sum_{n=1}^{\infty} sin(n) + tan(n)$ is divergent series.

Q 2). (a) Attempt any one from following. **(08)**

1) Let $f: [a, b] \to R$ be bounded function then f is integrable if and only if $\forall \varepsilon > 0$ $\exists P$ partition of $[a, b]$ such that $U(P, f) - L(P, f) < \varepsilon$

2) Let $f: [a, b] \to R$ be a bounded function an let P and Q be two partitions of [a,b] $P \subsetneq Q$ Then show that $L(P, f) \le L(Q, f)$

b) Attempt any two from following. **(12)**

1)Let $f: [a, b] \to R$ be a bounded function , is integrable if and only if there is sequence of partition (P_n) such that $\lim_{n\to\infty} [U(P_n, f) - L(P_n, f)] = 0$

2)Check integrability of function f(x) = 200 in [0,3] and evaluate f(x) = $\int_0^3 200\, dx$

3) Show that $f: [a, b] \to R = c$ (where c is constant) is Reimann integrable function.

Q 3). (a) Attempt any one from following. **(08)**

1)Let I=[α, β] and $\phi: I \to R$ be continuously differentiable function. If $f: [a, b] \to R$ Is a continuous function such that $\phi(\alpha) = a$ and $\phi(\beta) = b$ and $\phi(I) \subsetneq [a, b]$ then

$$\int_{\alpha}^{\beta} f(\phi(t) \cdot \phi'(t))\, dt = \int_a^b f(x)\, dx$$

2) Let $f, g: [a, b] \to R$ be differentiable function and f' and g' be Reimann integrable on [a,b] then show that $\int_a^b f(x)g'(x)\, dx = [f(b)g(b) - f(a)g(a)] - \int_a^b f'(x)g(x)\, dx$

b) Attempt any two from following. **(12)**

1)Find the surface area of solid of revolution obtained by rotating the curve x=3cos(θ)

Y=3sin(θ) , $0 \le \theta \le \pi$

2)Show that $\beta(m,n) = \beta(n,m) = \beta(m+1,n)$

3)$n \cdot \Gamma(n) = \Gamma(n+1), \quad n > 0$

Q 4). Attempt any three from following **(15)**

1) Apply suitable test to check convergence of $\sum_{n=1}^{\infty}(-1)^n \frac{3n-1}{2n+1}$

2) By Leibnitz test discuss convergence of $\frac{4}{7} - \frac{4}{8} + \frac{4}{9} - \frac{4}{10} + \cdots$

3) $f: [1,3] \to R$, where $f(x)$ is defined for $x \in [1,3]$ as f(x)=-2. Show that f(x) is integrable and evaluate it.

4)Prove that is *Let* $f: [a,b] \to R$ be a bounded function and let P be partition of [a,b] then show that $L(P,f) \le U(P,f)$

5) $Evaluate \int_0^{\infty} x^3 e^{-4x}\, dx$

6)Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

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