VCD/30/10/23 SYBSC- SEM III - MATHEMATICS III- 75MARKS- 2½HRS

NOTE : 1) All questions are compulsory.Use of Scientific Calculator is allowed.

2) For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any two subquestions (each 6marks) from part (b).

3) For Q.4 , attempt any three. (each 5 marks)

Q.1.(a)Attempt any one. [each 8Mks]

1) If the auxiliary equation of the nth order homogeneous linear differential equation $b_0y^n + b_1y^{n-1} + \cdots b_{n-1}y' + b_ny = 0$ …*has n distinct real roots $m_1, m_2, \ldots, m_n$ *then* prove that the general solution of (*)is given by $y = c_1e^{m_1x} + c_2e^{m_2x} + \cdots . + c_ne^{m_nx}$ *where* $c_1, c_2, \ldots c_n$ are arbitrary constants.

2)Let y_p be a particular solution to the non homogeneous linear differential equation $b_0(x)y^n + b_1(x)y^{n-1} + \cdots + b_{n-1}(x)y' + b_n(x)y = R(x)$ let y_c be the general solution to the corresponding homogeneous linear differential equation $b_0(x)y^n + b_1(x)y^{n-1} + \cdots + b_{n-1}(x)y' + b_n(x)y = 0$ then prove that $y = y_c + y_p$ is the general solution to the given non homogeneous linear system

(b)Attempt any two. [each 6Mks]

1) Let $f_1, f_2, \ldots, f_n$ be n functions defined on I =[a,b] where each of these functions is continuously differentiable atleast (n-1) times on I.If the Wronskian $W(f_1, f_2, \ldots, f_n)(x_0) \neq 0$ for some $x_0 \in I$ then prove that $f_1, f_2, \ldots, f_n$ are linearly independent.

2) Find the general solution to the differential equation $(9D^3 + 6D^2 + D)y = 0$

3) If $y_1, y_2, \ldots y_k$ are k solutions to nth order linear homogeneous differential equation $b_0(x)y^n(x) + b_1(x)y^{n-1}(x) + \cdots b_{n-1}(x)y'(x) + b_n(x)y = 0$ then Prove that $c_1y_1 + c_2y_2 + \cdots + c_ky_k$ is also a solution of given differential equation where $c_1, c_2, \ldots c_k \in R$

Q.2. (a)Attempt any one. [each 8Mks]

1) If w(t) is the Wronskian of two solutions $t \in [a, b], x = x_1(t), y = y_1(t)$ *and* $x = x_2(t), y = y_2(t)$ to the homogeneous linear system $\frac{dx}{dt} = a_1(t)x + b_1(t)y, \frac{dy}{dt} = a_2(t)x + b_2(t)y$ .....(*) of differential equations then prove that either w(t) is identically equal to zero or w(t) is never zero on [a,b]

2) Let $t \in [a, b], x = x_1(t), y = y_1(t)$ *and* $x = x_2(t), y = y_2(t)$ be two solutions to the homogeneous system of equations $\frac{dx}{dt} = a_1(t)x + b_1(t)y, \frac{dy}{dt} = a_2(t)x + b_2(t)y$ .....(*) then prove that $x(t) = c_1x_1(t) + c_2x_2(t), y(t) = c_1y_1(t) + c_2y_2(t)$ is also a solution to the above(*) homogeneous system of equations for t∈[a,b] where c_1, c_2 *are arbitrary constants.*

(b)Attempt any two. [each 6Mks]

1)Find the general solution to the following system $\frac{dx}{dt} = 4x - 3y, \frac{dy}{dt} = 8x - 6y.$

2)Prove that $x = 3t + 2, y = 2t - 1$ is particular solution to the non homogeneous linear system $\frac{dx}{dt} = x + y - 5t + 2, \frac{dy}{dt} = 4x - 2y - 8t - 8$,hence find general solution to above system.

3)Find general solution to the following system $\frac{dx}{dt} = x - 2y, \frac{dy}{dt} = 4x + 5y$.

Q.3. (a)Attempt any one. [each 8Mks]

1) Derive Picard's method formula to solve differential equation to find nth approximation for the differential equations $y' = f(x, y) with\, y(x_0) = y_0$.

2)Solve boundary value problem for $y'' - y = 0\ with\ y(0) = 0, y(1) = 1$ with h=0.5.

(b)Attempt any two. [each 6 Mks]

1) Apply Modified Euler's method to estimate the values using 2 approximation for $y' = 7xy^2 with\ y(0) = 1$and estimate y(0.25) taking step size h=0.25

2) Use Runge Kutta 2nd order method to approximate y at $x = 0.15$ $for\ the\ differential\ equation\ y' = \frac{7x-4y}{x+y^2}\ with\ y(0) = 1\ with\ h = 0.15$

3) For differential equation $y'' - 8y' + 7y = 10\ with\ y(0) = -1, y(1) = -3.\ estimate$ the value of y(0.5) taking the step size as h=0.5 using finite difference method.

Q.4.Attempt any three. [each 5 Mks]

1) Find the particular integral for $(D^2 + 4)y = \sin 2x$

2)Find the general solution to the following differential equation $(D^3 + 2D^2 - 15D)y = 0$

3)Find the equivalent system of first order equation for the initial value problem $x^4 \frac{d^4y}{dx^4} - 8x^3 \frac{d^3y}{dx^3} + 7x^2 \frac{d^2y}{dx^2} - 13x \frac{dy}{dx} + 8y = 0\ with\ y(0) = 1, y'(0) = 2, y''(0) = 3, y'''(0) = -6$

4)Show that $x = 3t - 2, y = -2t + 3$ is a particular solution to the non homogeneous system $\frac{dx}{dt} = x + 2y + t - 1, \frac{dy}{dt} = 3x + 2y - 5t - 2$

5) Evaluate $\sqrt{10} + \sqrt{11} + \sqrt{12}\ to\ 4$ significant digits and find absolute and relative error.

6) Use Taylor series method to find the value of $y\ at\ x = 0.1, \frac{dy}{dx} = 1 + xy\ given\ y(0) = 1$correct upto 4decimal places.

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