VCD/27/10/2023 SYBSC- SEM III - MATHEMATICS II- 75MARKS- 2½HRS

NOTE : 1) All questions are compulsory.

2) For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any two subquestions (each 6marks) from part (b).

3) For Q.4 , attempt any three. (each 5 marks)

Q.1. (a) Attempt any one. [each 8Mks]

1)Let A, B be two row equivalent matrices then prove that A is invertible iff B is invertible

2)Let A, B be the matrices of order m×n then prove that A, B are row equivalent off there existsan invertible matrix P such that B=PA

(b) Attempt any two. [each 6Mks]

1)Check whether the following system of equation is consistent and if so, find the solution set. $x_1-x_2+2x_3=1$, $2x_1+2x_3=1$, $x_1-3x_2+4x_3=2$

2) Check whether the following matrices are elementary matrices

$i)\begin{bmatrix}0 & 0 & 1\\0 & 1 & 0\\1 & 0 & 0\end{bmatrix} ii)\begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix} iii)\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

3)Express the following matrices and their inverses as product of elementary matrices $\begin{bmatrix}9 & 11\\6 & 2\end{bmatrix}$

Q.2. (a) Attempt any one. [each 8Mks]

1)Let V be a real vector space and W be a subspace of V then prove that

i) v+W=W iff v belongs to W ii) $v_1+W=v_2+W$ iff v_1-v_2 belongs to W

2) Define Linearly independent set and Prove that Superset of linearly dependent set is linearly dependent

(b) Attempt any two. [each 6Mks]

1)Check whether the given vector v belong to L(S) linear span of S in the following space

v=(1, 3,2), S={(2,1,-1),(3,2,0),(4,0,3)} in R^3

2) Check whether $S_1 = \{(-1,1),(3,2),(4,6)\} in\ R^2$ are Linearly independent or not?

3)Prove that (R,+,*)is a real vector space with respect to usual addition+ and multiplication*.

Q.3. (a) Attempt any one. [each 8Mks]

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1) Let A be A n×n real matrix.If det(A) ≠0 then prove that A is invertible and $A^{-1} = \frac{1}{\det A}(Adj\ (A))$

2) Prove that $\det(V_n) = \det(V_n^T) = \prod_{1\le i<j\le n} (x_i - x_j)$

(b) Attempt any two. [each 6 Mks]

1)Find row rank, column rank and rank of $\begin{bmatrix} 0 & 2 & 6 & 5 & 9 \\ 1 & 2 & 3 & 6 & 2 \\ 2 & 4 & 0 & 1 & 0 \end{bmatrix}$ using row echelon form Also find basis of row space and column space.

2)Prove that if A is a square matrix then $\det(A) = \det(A^t)$

3)Find A^{-1} where $A = \begin{bmatrix} 4 & 6 & -5 \\ -3 & -2 & 5 \end{bmatrix}$ using adjoint of A.

Q.4.Attempt any three. [each 5 Mks]

1)Show that the following system of equations have infinitely many solutions applying Gauss Elimination method 2x-y+3z=5, 4x+10y+z=14

2)Prove that the following system of equations have no solutions.
x-3y+8z=11,2x+y-7z=9,-7y+23z=5,7x+7y-36z=10

3) Check whether Q the set of rational number are vector space over R the set of real numbers with respect to usual addition and multiplication.

4) Prove that $S = \{(1,0), (0,1)\}$ is a basis of R^2.

5)Find the determinant of $\begin{bmatrix} -1 & 3 & 2 \\ 4 & 6 & -5 \\ -3 & -2 & 5 \end{bmatrix}$ using Laplace expansion along colmn 3

6)Solve AX = B using LU decomposition method where $A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & 1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$ $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 6 \\ 3 \end{bmatrix}$

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