VCD/170423 FY B.Sc (PCM/PMS) Sem-II Maths-II(Discrete Mathematics)75 Marks-2.5hrs

Note: (i) All questions are compulsory.
(ii) For Q1, Q2 and Q3 attempt any one sub-question (each 8 marks) from part (a) and any two sub-questions (each 6 marks) from (b) part.
(iii) For Q4, attempt any three (each 5 marks).

Q1. (a) Attempt any one: [each 8 marks]

1. Let A be a non-empty set.
   (i) A is countable.
   (ii) There exists an injective function f: $A \to \mathbb{N}$
   (iii) There exists a surjective function g: $\mathbb{N} \to A$
   Prove that (i) implies (ii) and (ii) implies (iii).
2. (i) Prove that $\mathbb{Z}$ (set of integers) is countable.
   (ii) Give two examples of uncountable sets.

(b) Attempt any two: [each 6 marks]

1. Prove that subset of a finite set is finite.
2. In how many ways can we draw a heart or an ace from a deck of 52 playing cards? A numbered card or a king? A red card or a queen?
3. If A and B are countable sets, then prove that A X B is also countable.

Q2. (a) Attempt any one: [each 8 marks]

1. (i) State Binomial theorem.
   (ii) State and prove Pascal's identity.
2. Prove: (i) $\binom{2n}{2} = 2\binom{n}{2} + n^2$
   (ii) $\binom{n}{k}\binom{k}{m} = \binom{n}{m}\binom{n-m}{k-m}$

(b) Attempt any two: [each 6 marks]

1. Find the no. of integers between 1 and 1000, inclusive, that are divisible by 5, 6 or 8.
2. Evaluate $\varphi(276)$ *and* D_4 where φ represents Euler's function and D_n represents number of derangements on n symbols.
3. Let S be a multi-set with k types of objects with repetition numbers $n_1, n_2, ... n_k$ with $n_1 + n_2 + ... + n_k = n$, then prove that number of permutations of S is given by

$$\binom{n}{n_1\, n_2 \ldots .\, n_k} = \frac{n!}{n_1!\, n_2! \ldots n_k!}$$

Q3. (a) Attempt any one: [each 8 marks]

1. (i) Prove that every permutation in S_n can be expressed as product of disjoint cycles.
   (ii) List all elements of S_3.

2. Suppose the characteristic equation $x^2 - \alpha x - \beta = 0$ of the recurrence relation $a_n = \alpha a_{n-1} + \beta a_{n-2}$ for $\alpha, \beta \in \mathbb{R}$ has 2 real and equal roots q and q, then prove that the general solution of recurrence relation is given by $a_n = c_1 q^n + c_2 n q^n$ where c_1 *and* c_2 are arbitrary constants.

(b) Attempt any two: [each 6 marks]

1. Solve the recurrence relation: $a_n = a_{n-1} + 6a_{n-2}$, $a_0 = 3, a_1 = 6$.

2. Prove that the number of even permutations on n symbols is equal to the number of odd permutations on n symbols for n > 1.

3. Let $\alpha = \begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 3\,1\,4\,6\,2\,5 \end{pmatrix}$ *and* $\beta = \begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 6\,4\,5\,1\,3\,2 \end{pmatrix}$ *in* S_6, then show that

$(\alpha o \beta)^{-1} = \beta^{-1} o \alpha^{-1}$.

Q4. Attempt any three: [each 5 marks]

1. How many minimum numbers must be selected from the set {1,2,3….,11} to guarantee that two of the selected numbers give sum as 12?
2. Find S(5,2) and S(5,4) using recurrence relation where S(n, k) represents Stirling number of second kind.
3. Find number of terms in the expansion of $(3x - 4y + 8z + 6w)^9$.
4. Ten people, including two who do not wish to sit next to one another, are to be seated at a round table. How many circular seating arrangements are there?
5. Solve recurrence relation $a_n = a_{n-1} + n|$, $a_0 = 4$ using iterative method.
6. Find the signature of the following permutation in S_5 using definition:

   $\alpha = \begin{pmatrix} 1\,2\,3\,4\,5 \\ 5\,3\,2\,4\,1 \end{pmatrix}$