VCD/190423FYBSC SEM II STATISTICAL METHODS-II 3 HRS 100 MARKS

N.B.: (1) All questions are compulsory.

(2) Use of Scientific Non-programmable Calculator is allowed.

Q.1 (A) State True or False and correct if necessary. **(10)**

(1) If $f(x)$ is a probability density function (p.d.f.) of a continuous random variable X, then $\int_{-\infty}^{\infty} f(x) = 0$.

(2) If F(x) is a cumulative distribution function (c.d.f.) of a continuous random variable X, then a < b implies F(a) < F(b).

(3) If X ~ U(a,b), then $E(X)=\frac{a-b}{2}$.

(4) The mean of exponential distribution with parameter λ is $\frac{1}{\lambda}$.

(5) If in a decision criteria, $p > p_0$, reject H_0 when $Z_{cal} > Z\alpha$.

(B) Answer in one sentence. **(10)**

(1) State the formula for the variance of a continuous random variable X.

(2) If X ~ U(4,12), then what is the mean of X?

(3) Define exponential distribution with parameter λ.

(4) Write the formula for margin error.

(5) What are the two types of statistical inference?

Q.2 Attempt any TWO sub-questions: **(20)**

(1) Write the properties of probability density function (p.d.f.) and verify whether following functions are p.d.f. **(10)**

(a) $f(x) = nx^{n-1}$ $, for\ 0 < x < 1$

$= 0$ $, otherwise$

(b) $f(x) = 5e^{-4x}$ $, for\ x \geq 0$

$= 0$ $, otherwise$

(2) If $f(x) = 12x^2(1-x)$ $, for\ 0 < x < 1$

$= 0$ $, otherwise$ **(10)**

Obtain its c.d.f. and also find $P(x < 0.3)$, $P(x \geq 0.45)$, $P(0.25 < x < 0.60)$.

(3) Let X be a continuous random variable with p.d.f. given by, **(10)**

$f(x) = 3(1-x)^2$ $, for\ 0 < x \leq 1$

$= 0$ $, otherwise$

Obtain (a)c.d.f. (b)first quartile (c)second quartile (d)second decile (e)70th percentile .

Q.3 Attempt any TWO sub-questions: (20)

(1) Define uniform distribution over an interval (a,b). Derive its mean and variance. (10)

(2) Studies of a single machine tool system showed that the time, the machine operates before breaking down is exponentially distributed with a mean 10 hours. (10)

(i) Find the probability that the machine operates for (a)at least 12 hours before breaking down (b)at least 14 hours but fails before 20 hours.

(ii) If the machine has already been operating 8 hours, what is the probability that it will last another 4 hours?

(3) If continuous random variable X follows normal distribution with mean 50 and variance 100, then find (a)$P(X \le 70)$ (b)$P(X > 65)$ (c)$P(X \le 30)$ (d)$P(35 < X < 60)$ Given that $\phi(2)=0.9772$, $\phi(1.5)=0.9332$, $\phi(1)=0.8413$ (10)

Q.4 Attempt any TWO sub-questions: (20)

(1) Write the 6 step procedure of testing hypothesis. (10)

(2) An investor has developed a new energy efficient lawn mower engine. He claims that the engine will run continuously for 5 hours (300 minutes) on a single gallon of regular gasoline. From his stock of 2000 engines the inventor selects a simple random sample of 50 engines for testing. The engines run for an average of 295 minutes with standard deviation of 20 minutes. Test the inventor's claim at 0.05 level of significance. (10)

(3) Find the probability that of the next 120 births, no more than 40% will be boys. Assume equal probabilities for the births of boys and girls. Assume also that the number of births in the population (N) is very large, essentially infinite. (10)

Q.5 Attempt any FOUR sub-questions: (20)

(1) Let X be a continuous r.v. with p.d.f. given by
$$f(x) = cx^2 \quad , for\ |x| \le 1$$
$$= 0 \quad , otherwise$$

Find (a)constant c (b)$P(X > 1/2)$ (5)

(2) Let X be a continuous random variable with p.d.f.
$$f(x) = \frac{x+1}{2} \quad for\ -1 < x < 1$$
$$= 0 \quad \text{otherwise}$$

Obtain c.d.f of X. What is 64th percentile of X? (5)

(3) If $X \sim U(-1,3)$ and $Y \sim Exp(\lambda)$, find λ such that $V(X) = V(Y)$. (5)

(4) If $X \sim N(10,16)$ and $Y \sim N(5,25)$, where X and Y are independent, then obtain
(a)distribution of $W=X+Y$ (b)distribution of $W=X-Y$ (5)

(5) A sample of 200 people has a mean age of 21 years with population standard deviation (σ) of 5. Test the hypothesis that the population mean is 18.9 years at $\alpha = 0.05$. (5)

(6) A principal at a certain school claims that the students in his school are above average intelligence. A random sample of thirty students IQ scores has a mean score of 112.5. Is there sufficient evidence to support the principal's claim? The mean population IQ is 100 with a standard deviation of 15. (5)