VCD/150423 FYBSC- SEM II - MATHEMATICS I- 75MARKS- 2½HRS

NOTE : 1)All questions are compulsory.

2)For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any two subquestions (each 6marks) from part (b).

3)For Q.4 , attempt any three. (each 5 marks)

Q.1. (a)Attempt any one. [each 8Mks]

1) Given that $\lim_{x\to a} f(x)\ \lim_{x\to a} g(x), a \in R$ exist then prove that

$$\lim_{x\to a}\big(f(x).g(x)\big) = \lim_{x\to a} f(x).\lim_{x\to a} g(x)$$

2) State and prove Sandwich Theorem for limit of a function

(b)Attempt any two. [each 6Mks]

1) Using ∈-δ definition of limit, prove the limit of f(x)=3x+1 at x=2 is 7

2)$\lim_{x\to a} f(x) = l$ then prove that f(x) is bounded in some deleted neighbourhood of a.

3) Prove that $x^2-2=0$ is solvable in R

Q.2. (a)Attempt any one. [each 8Mks]

1)Define the differentiability at point p of f:I→R Where I is an open interval in R,p∈R Prove that if f is differentiable at p ∈I then it is continuous at p.

2) State and prove Inverse Function Theorem

(b)Attempt any two. [each 6Mks]

1)If $f: I \to R, g: I \to R$ are differentiable at p∈I ,Where I is an open interval in R then prove that fg is differentiable at p and $(fg)'(p) = f'(p)g(p) + f(p)g'(p)$

2) Define the left hand derivative of f and right hand derivative of f at p ∈I for f:I→R Where I is an open interval in R and find Df(0-), Df(0+) for f:R→R as f(x)=0. x≤0

=x^2. x>0. at x=0

3) State Leibnitz rule to find nth derivative of product of two functions,find for $y = x^3e^x$

Q.3 (a)Attempt any one. [each 8Mks]

1) State and prove Rolle's Mean Value Theorem for real valued function f.

munotes.in

Scanned with OKEN Scanner

2) State and prove Lagrange's Mean Value Theorem for real valued function f.

(b)Attempt any two. [each 6 Mks]

1)Define local minimum and local maximum value for real valued function f on (a,b) and show that $f(x) = (x-1)^2, x \in R$ has local minimum value 0 at x=1.

2) Verify Cauchy's mean value theorem for the functions x^2&x^3on [1,2]

3) Using L'Hospital's Rule , $\lim_{x \to 0} \frac{x^2+2\cos x-2}{x \sin^3 x}$

Q 4.Attempt any three. [each 5 Mks]

1)$\sqrt{5-2x^2} \leq f(x) \leq \sqrt{5-x^2}$ $for -1 \leq x \leq 1$ $then\ find \lim_{x \to 0} f(x)$

2) Evaluate $\lim_{x \to \infty} \frac{3x^2+2x-5}{6x^2+8x-3}$

3)If y=cos(ax+b) then prove that $y_n = a^n \cos\left(x + b + \frac{n\pi}{2}\right)$

4) Find $\frac{dy}{dx}$ if $\sin(x+y) = e^{xy}$

5)Prove that Maclaurin's series expansion for $f(x) = e^x$ $is\ 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

6) Sketch the curve $y = \frac{x^2+1}{x^2-1}$

munotes.in

XXXXXXXX

Scanned with OKEN Scanner