

NOTE : 1) All questions are compulsory.

2) Use of calculator is allowed.

Q.1. (a) State true and false, with justification.

[Each 2 marks]

1. A discrete random space must contain a finite number of elements.
2. If A' and B are independent, then A and B are independent.
3. Kurtosis explains peakedness.
4. With usual notation $Cov(aX + b, cY + d) = ac Cov(X, Y) + bd$.
5. Poisson distribution is always negatively skewed.

(b) Answer in one line.

[Each 2 marks]

1. Give an example of (i) mutually exclusive (ii) exhaustive event.
2. State multiplicative theorem for three events.
3. Define probability mass function of random variable X .
4. Define covariance between two random variable.
5. Define bernoulli experiment.

Q.2 Attempt any TWO.

[Each 10 marks]

- (1) State and prove Bayes' theorem.
- (2) (a) Explain the concept of conditional probability.
(b) Define pair-wise independence and total independence.
- (3) (a) Suppose a card is drawn from a well shuffled pack of cards. Let event A : getting heart card and B : getting queen. Are A and B independent?
(b) A husband and a wife appear for two vacancies in the same post. The probability of husbands selection is $1/6$ and wife's selection is $1/5$, what is the probability that
(i) both will be selected (ii) only one of them would be selected (iii) none of them would be selected?

Q.3 Attempt any TWO.

[Each 10 marks]

- (1) Define Cumulative distribution and its Properties. And prove $E(aX+b) = aE(X) + b$.
- (2) Define covariance between two random variables. State and prove its properties.
- (3) If bivariate probability mass function is given by

$$P_{X,Y}(x, y) = \begin{cases} K(2x + y) & x, y = 2, 3, 4 \\ 0 & \text{Otherwise} \end{cases}$$

Obtain the value of K . Also obtain marginal p.m.f. of X and Y . and conditional p.m.f. of X given $Y = 2$.

Q.4 Attempt any TWO.

[Each 10 marks]

- (1) (a) If a random variable X follows binomial distribution with parameters (n, p) , Obtain expression for its mean & variance.

- (b) If a random variable X follows Poisson distribution with parameters λ , Obtain expression for its mean & variance.
- (2) (a) If a discrete random variable X has uniform distribution over the range $\{1, 2, 3, \dots, n\}$. And its mean is 12.5 find n and its variance.
 (b) For binomial variate X with parameter (n, p) ; $p = q$ and $P(X=2) = P(X=3)$, Find (i) $P(X=1)$ (ii) $P(X > 1)$
- (3) (a) If random variable X follows Poisson distribution, if $P(X=5) = 0.3 P(X=4)$ then find mean and $P(X > 3)$.
 (b) A small voting district has 101 female voters and 95 male voters. A random sample of 10 voters is drawn. What is the probability that exactly 7 of the voter will be female? (using hypergeometric distribution).

Q.5 Attempt any TWO.

[Each 10 marks]

- (1) State and prove the addition theorem of probability concerning 2 events A and B .
- (2) (a) If bivariate p.m.f is given by

$$P_{X,Y}(x, y) = kxy \quad x, y = 1, 2, 3$$

$$= 0 \quad \text{otherwise}$$

Obtain the value of k . also obtain marginal probabilities of X and Y . Are X and Y independent?

- (b) Consider the experiment of tossing four unbiased coins. The random variable X is number of heads observed and Y be the number of heads observed in a sequence. Then write down joint probability mass function of X, Y . Hence their marginal probability, also obtain conditional p.m.f. of X given $Y = 2$.
- (3) (a) An unbiased dice is rolled. Write down the p.m.f for the number on the uppermost face. Obtain its mean and variance.
- (b) A box contain 10 balls of which 6 are red and 4 are white. Two balls are selected at random. If X denotes number of white balls selected then write down p.m.f. of X and also state mean and variance of X .
