VCD/ 9|12|23 **FYBSC SEM I- MATHEMATICS II- 75MARKS- 2½HRS**

NOTE: 1) All questions are compulsory.

2) For Q.1, Q.2 and Q. 3 attempt any one sub-question (each 8 marks) from part (a), and any two sub-questions (each 6marks) from part (b).

3) For Q.4, attempt any three. (each 5 marks)

Q 1). (a) Attempt any one from following. (08)

1) State and prove binomial theorem for $n \in N$ where N is set of natural numbers.

2) Prove that for given integer a and b , $b > 0$, there exist unique integers q

And r such that $a = bq + r$, $0 \leq r < b$

b) Attempt any two from following. (12)

1) State Euler's Phi function and calculate $\phi(1001)$ and $\phi(360)$

2) If $(a, b) = 1$ and $c|\ (a + b)$ then prove that $(a, c) = (b, c) = 1$, where (a, b) is g.c.d of a,b.

3) Using mathematical induction prove that $n(n + 1)$ is divisible by 2.

Q2). (a) Attempt any one from following. (08)

1) $f: X \to Y$ and $g: Y \to Z$ are two function such that $g \circ f$ (composition function)is bijective

i. f is surjective, then prove that g is injective.

ii. g is injective, then prove that, f is surjective.

2) define binary operation, Commutativity, associativity, existence of identity element, existence of inverse element. Also check all properties for $a + b, a, b \in R, R$

Is set of real numbers.

b) Attempt any two from following. (12)

1) Check whether relation R defined as $aRb, iff\ a \leq b, \in R\ for\ a, b \in R(set\ of\ real\ numbers)$ is equivalence relation.

2) Find inverse of $f(x) = e^x$ in $R \to R^+$ and $f(x) = log(x)$ in $R \to R^+$

3) Show that the function $f: R \to R, f(x) = 2x + 5$ is bijection.

Q 3). (a) Attempt any one from following. (08)

1) State and prove Remainder theorem for $R[x]$, Also show that A polynomial $f(x)$ in $R[x]$.

Is divisible by $(x - a)$ iff $f(a) = 0$, where $R[x]$ is set of polynomials with real coefficients.

2) Prove that every Polynomial of degree n, with $n > 2$ is reducible in $R[x]$.

b) Attempt any two from following. (12)

1) If p is a positive prime number, then prove that $\sqrt{p}$ is an irrational number.

2) Find multiplicity of roots of $f(x) = 4x^3 + 4x^2 - x - 1$

3) Find fourth roots of unity.

Q 4). Attempt any three from following (15)

1) Show that $555^{661} \equiv 1\ mod(105)$

2) Verify Wilson's theorem for $p = 7$

3) $a * b = 4a - 5b, a, b \in Z$, test commutativity and associativity of '*' .

4) Prove that $f: R \to R$ is given by $f(x) = 5x - 8$ is bijection.

5) Find g.c.d of polynomials, $f(x) = x^8 - 1, \quad g(x) = x^6 - 1$.

6) Prove that multiplication is binary operation in $R[x]$, Where $R[x]$ is set of real polynomials.

********** ALL THE BEST *********