8|12|23

VCD/ FYBSC- SEM I - MATHEMATICS I- 75MARKS- 2½HRS

NOTE : 1) **All questions are compulsory.**

2) **For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any two subquestions (each 6marks) from part (b).**

3) **For Q.4 , attempt any three. (each 5 marks)**

Q.1. (a)Attempt any one. [each 8Mks]

1) State the properties of real numbers with respect to addition`+`

2)Let S be a nonempty subset of R real numbers then prove that a real number M is supremum of S iff i)$x \le M \forall x \in S$ ii)$for\ every \in > 0 \exists\ x \in S\ such\ that\ M - \in < x \le M$

(b)Attempt any two. [each 6Mks]

1)State and prove Hausdorff Property of real numbers R.

2)State and prove Archimedean Property of real numbers R.

3)Prove that i)Every real number has a unique Additive inverse.ii)Additive identity in R is unique.

Q.2. (a)Attempt any one. [each 8Mks]

1)Prove that $\left(1 + \frac{1}{n}\right)^n$ is convergent.

2)Prove that Every monotonic increasing sequence of R converges to its l.u.b(least upper bound) if bounded above.

(b)Attempt any two. [each 6Mks]

1)Prove that a convergent sequence of real numbers R has a unique limit .

2) Prove that every convergent sequence of Real numbers R is bounded.What about the converse? Justify.

3)State and prove Sandwich Theorem for limit of a sequence in R.

Q.3. (a) Attempt any one. [each 8Mks]

1)Prove that The differential equation $\frac{dy}{dx} + P(x)y = Q(x)y^n$where P(x), Q(x)are functions of x and n is real number, n ≠

1, is always reducible to a suitable Linear Differential Equation, of the type $\frac{dv}{dx} + P_1(x)v = Q_1(x)$.

2)State Rule I to find Integrating factor and solve the following differential equation $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

(b) Attempt any two. [each 6 Mks]

1)A bacteria culture is known to grow at a rate proportional to the amount present.After one hour,1000 bacteria are observed in the culture, and after four hours 3000.Find an expression for the number of bacteria present in the culture at any time t.Also determine the number of bacteria originally in the culture.

2)Find Orthogonal trajectories of $y = cx^2$

3) Solve the following linear differential equation $\frac{dy}{dx} + \frac{y}{x} = 5x^3 - 6x + 8$

Q.4.Attempt any three. [each 5 Mks]

1)If a, b, c $\in R^+$ then prove that $(a + b)(a + c)(b + c) \geq 8abc$

2) Write the expression $|5 - x^{-1}| \leq 1$ in the interval form.

3) Examine whether the following sequences are monotonic $x_n = \frac{4}{n+1}, n \in N$

4) Use Sandwich Theorem to show following sequence is convergent $x_n = \frac{1}{3^n}$ *where* $n \in N$.

5) Solve the Bernoulli's differential equation $\frac{dy}{dx} + xy = e^{x^2}y^3$

6) Solve the differential equation $(x^2 - y)dx - xdy = 0$

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