FYBSC sem I
F.y.BSC - MATHS I
sem - I
25/03/2015

VCD - 250315 MATHS I - FYBSC - SEM I - EXAM - 75 MARKS - 2.5HRS - 250

NOTE : 1) All questions are compulsory.

2) For Q.1, Q.2 and Q. 3 attempt any one subquestion (each 8 marks) from part (a), and any three subquestions (each 4 marks) from part (b).

3) For Q.4 , attempt any three.(each 5)

Q.1. (a) Attempt any one. [each 8]

1) For $a, b, c \in IR$ prove the following

i) $a < b \Rightarrow -a > -b$

ii) $a < b \Rightarrow a + c < b + c$

iii) $a > b \,\&\, c < 0 \Rightarrow a < c$

iv) $a > b \,\&\, c > 0 \Rightarrow ac < bc$

2) State all properties of addition of IR = set of real numbers.

(b) Attempt any three. [each 4]

1) Find LUB and GLB of $S = \{ {}^{1}/_{n} / n \in IN\}$

LUB= Least Upper Bound

GLB = Greatest Lower Bound

2) State and prove AM – GM inequality of IR.

3) State and prove Hausdorff property of neighbourhood.

4) Prove that if a_n is convergent sequence then it is bounded.

Q.2. (a) Attempt any one. [each 8]

1) State and prove Sandwhich theorem for sequence.

2) State all algebraic properties of sequences.

(b) Attempt any three. [each 4]

1) Prove that i) if $a_n \to a \,\&\, \alpha \in IR$ then $\alpha . a_n \to \alpha a$.

2) Prove that if $a_n \neq 0$ $for\ any\ n \in IN$ then $a_n \to a \Rightarrow \frac{1}{a_n} \to \frac{1}{a}$.

3) Show that $a_n \to a$ then $|a_n| \to |a|$.

4) Define monotonic increasing and decreasing sequence and give example

PTO

Q.3. (a) Attempt any one. [each 8]

1) Let f, g be real valued function defined on subset $J \subseteq IR$ and $\lim_{x \to a} f(x) = l$, $\lim_{x \to a} g(x) = m$ then prove that $\lim_{x \to a} f(x) \cdot g(x) = l.m.$

2) Let $f : I \to IR$ be a function where I is an open interval in IR. Let $a \in I$ then prove that $\lim_{x \to a} f(x)$ exist iff $\lim_{x \to a+} f(x) = \lim_{x \to a-} f(x)$

Q.3. (b) Attempt any three. [each 4]

1) Define the following functions and draw the graphs

i) Constant function

ii) Flooring function $\lfloor x \rfloor$

2) Define right hand and left hand limits and evaluate right hand left hand limits of f as $\lim_{x \to 3} f(x)$ in following case

$$f(x) = x - 3 \qquad if\ (x-3) \geq 0$$

$$= -(x-3) \qquad if\ (x-3) < 0 \quad p = 3$$

3) Let $f, g : J \to IR$ be continuous at $P \in J$ where J is an open interval in IR then prove that $f + g : J \to IR$ is continuous at P.

4) Give $\epsilon - \delta$ definiton of limit of f at P and show that $\lim_{x \to 4}(2x + 3) = 11$.

Q.4. Attempt any three. [each 5]

1) Show that $f: IR \to IR$ defined by $f(x) = c \ \forall x \in IR$, c is constant is continuous everywhere.

2) Let f be real valued defined on subset $I \subseteq IR$. Let $P \in I$, if $\lim_{x \to P} f(x)$ exists then prove that it is unique.

3) Prove that $\lim_{n \to \infty} \frac{a^n}{n!} = 0 \quad \forall a \in IR$.

4) Prove that $x \in IR^+$ then $\exists\, m \in IN$ such that $0 < \frac{1}{m} < x$.

5) Examine whether the following sequences are monotonic

i) $a_n = \frac{4}{n+1}$ ii) $a_n = \frac{4+3n}{n}$

6) Prove that if a_n & b_n are two convergent sequences having limits a and b respectively then prove that $(a_n + b_n)$ converges and $\lim_{n \to \infty}(a_n + b_n) =$ $= a + b$.