

Note: (i) All questions are compulsory.  
(ii) Use of Calculator is allowed.

**Q.1) Answer the following questions**

**a) Correct the following if necessary:**

**(10M)**

- i. M.G.F. uniquely determines the distribution.
- ii. Binomial distribution is always unimodal.
- iii. All Cumulants are unequal in case of Poisson distribution.
- iv. Hypergeometric distribution has 2 parameters.
- v. If  $E(Y/X) = E(Y)$  then X and Y are independent.

**b) Answer in One sentence:**

**(10M)**

- i. Write down p.m.f. of discrete uniform variate over the range  $\{1, 2, \dots, n\}$  and also state its mean and variance.
- ii. If the moment generating function is given by  $M_X(t) = [0.55 + 0.45e^t]^{10}$ . Write down its probability mass function and mean.
- iii. Write down the p.m.f of poisson variable and write the expression for mean of poisson distribution.
- iv. Derive the expression for M.G.F. of Geometric Distribution.
- v. Define conditional probability density of X for given values of Y.

**Q.2) Attempt any TWO**

**(20M)**

**a) (1) If X is a r.v. with M.G.F.  $M_X(t)$ . Then prove that**

**07**

- (i) M.G.F. of  $Y = X+b$  is  $M_Y(t) = e^{bt}M_X(t)$
- (ii) M.G.F. of  $Y = aX+b$  is  $M_Y(t) = e^{bt}M_X(at)$
- (iii) If X and Y are two independent variables with their respective M.G.F.  $M_X(t)$  and  $M_Y(t)$ , then M.G.F. of  $X+Y$  is  $M_{X+Y}(t) = M_X(t) \cdot M_Y(t)$
- (iv)  $M_X(0) = 1$

**(2) If r.v. X has p.d.f.**

**03**

$$f(x) = \frac{1}{4} \quad ; -2 < x < 2$$

$$= 0 \quad ; \text{otherwise}$$

**P.T.O.**

Show that its M.G.F. is given by

$$M_X(t) = \frac{e^{2t} - e^{-2t}}{4t}, \quad t \neq 0$$

$$= 1, \quad t = 0$$

b) If r.v. X has p.m.f.

$$P(X=x) = \frac{x}{10} \quad ; x=1,2,3,4$$

$$= 0 \quad ; \text{otherwise}$$

Obtain its M.G.F. Find E(X) and V(X). State M.G.F. of  $Y=5X-3$ . Find E(Y).

- c) If a random variable X follows Bernoulli Distribution with probability p.  
Obtain expression for its moment generating function. Hence evaluate its mean, variance and measure of skewness.

**Q.3) Attempt any TWO**

**(20M)**

- a) (i) State and prove the Memory Loss Property for Geometric Distribution with parameter p. 07  
(ii) If a random variable X follows Poisson distribution if  $P(X=5) = P(X=4)$   
Then find SD and mode. 03

- b) Prove that the sum of two independent Poisson variates is a Poisson variate while the difference is not a Poisson variate.  
c) Evaluate the mean and variance using M.G.F. for Geometric function.

**Q.4) Attempt any TWO**

**(20M)**

- a) Define marginal and conditional probability mass function. derive conditional probability mass function.  
b) (i) State and prove additional theorem on Expectation of two discrete random variables.  
(ii) Multiplication theorem on Expectation of two discrete random variables.

P.T.O.

c) For the following joint p.m.f of X, Y

$\begin{matrix} \rightarrow Y \\ X \downarrow \end{matrix}$	1	2	3
0	0.02	0.08	0.10
1	0.03	0.12	0.15
2	0.05	0.20	0.25

(i) Examine whether X and Y are independent?

(ii) Find  $P[X+Y \leq 3]$

**Q.5) Attempt any TWO**

**(20M)**

- a) (i) With usual notations, if  $\phi_X(t) = (2 - e^{it})^{-1}$ . Write its M.G.F. Find  $E(X)$  and  $V(X)$ ..
- (ii) For a binomial variate mean is 6 and variance is 4, find (1)  $P(X=1)$  (2)  $P(X>2)$
- b) The number of monthly breakdown of a computer is a random variable having Poisson distribution with mean 1.8. Find the probability that this computer will work for a month
- with only one defective
  - with at least 2 defective
- c) If X and Y are two discrete random variables with  $E(X) = 10$ ,  $V(X) = 20$ ,  $V(Y) = 16$  and  $\text{Cov}(X, Y) = 2$  Obtain: (i)  $E(4X + 2Y + 5)$ , (ii)  $V(3X + 2Y)$ , (iii)  $V(4X - Y + 2)$ , (iv)  $\text{Cov}(2X + 1, 3Y + 2)$ .