

NOTE: 1) All questions are compulsory.

2) Use of calculator is allowed.

Q.1) Answer the following questions:

a) Correct the following if necessary:

(10M)

- (i) If A and B are mutually exclusive events, then $A \cup B = \emptyset$.
- (ii) Events A and B are independent iff $P(A/B) = P(A)$.
- (iii) A discrete random variable takes values $-2, -1, 0.5, 2.5$.
- (iv) The second raw moment about origin is variance.
- (v) Mean and standard deviation of discrete uniform distribution are equal.

b) Answer in one sentence:

(10M)

(i) If $P(A \cup B) = \frac{17}{30}$, $P(B) = \frac{2}{5}$ and $P(A \cap B) = \frac{1}{6}$, find $P(A)$.

(ii) Write down the classical definition of probability.

(iii) If probability mass function of X is given below find $P(X < 2.3)$

X	1	2	3	4
P(X=x)	1/4	3/8	1/4	1/8

(iv) If X and Y are independent random variables then what is $\text{COV}(X, Y)$

(v) State p.m.f of binomial distribution.

Q.2) Attempt any two from the following:

(20M)

- (i) State and prove the addition theorem on probability of two events. Also prove that if A and B are two events defined on sample space S, then $P(A \cup B) \leq P(A) + P(B)$.
- (ii) From a well shuffled pack of cards, 2 cards are selected at random, what is the probability that
 - a) both cards are red b) one is red and other is black c) both are aces
 - d) exactly one jack is drawn e) both are of the same suit
- (iii) Define following events and give example for each.
 - (a) Certain event (b) Mutually exclusive event (c) Complementary events
 - (d) Impossible event (e) Elementary/Simple events

Q.3) Attempt any two from the following:

(20M)

- (i) Define Random Variable, Discrete Random Variable, Probability Mass Function. Hence find i) $P(X \leq 1)$ ii) $P(X \geq -2)$ also obtain the probability distribution of $Y = X^2$ where probability mass function of a discrete random variable X is given by

X	-2	-1	0	1	2
P(X=x)	0.1	0.15	0.2	0.15	0.4

- (ii) Let X be a discrete random variable with probability mass function $p(x)$. a and b are constants. Then prove i) $E(aX + b) = aE(X) + b$ ii) $V(X) = E(X^2) - [E(X)]^2$. Also derive expression for 3rd central moment.
- (iii) State and prove Multiplication Theorem of Expectation.

Hence obtain i) the marginal probability mass function of X .
 ii) the marginal probability mass function of Y . For the joint p.m.f of random variables X and Y is given by

$$P(x, y) = \frac{(x+y)}{21} \quad x=1, 2, 3; \quad y=1, 2.$$

$$= 0 \quad \text{otherwise.}$$

Q.4) Attempt any two from the following: (20M)

- (i) Let X be a random variable having Uniform Discrete Distribution assuming values $\{1, 2, \dots, N\}$. Derive the expression for its mean and variance.
- (ii) (a) Derive the recurrence relation between probabilities for a random variable X having Binomial Distribution. (07)
 (b) For a Binomial Distribution mean is 3 and $15P(X \geq 0) = 2P(X \geq 1)$. Find $P(X = 5)$. (03)
- (iii)(a) Write the p.m.f. of Hyper-geometric Distribution with parameters (N, M, n) and derive the expression for mean of Hyper-geometric Distribution.
 (b) If discrete random variable X has binomial distribution with parameters $(6, p)$. Find p and q if $9P(X = 4) = P(X = 2)$.

Q.5) Attempt any two from the following: (20M)

- (i) A) A family has 2 children. Find the probability that both children are girls if it is known that:
 (a) one of the children is girl.
 (b) the older child is a girl.
 B) A ticket is drawn from the box containing 25 tickets and a number on it is observed. Obtain the probability that ticket drawn has a number
 (a) less than 6 (b) greater than 20
 (c) multiple of 5 (d) lying between 10 and 15, both inclusive
- (ii) Define Bivariate Discrete Distribution, Marginal Probability Distribution, Conditional probability Distribution. X and Y are 2 random variables with joint probability distribution function as given below. Find a marginal probability mass function of X and Y also obtained conditional probability distribution of X when $Y = 6$ and of Y when $x = 2$

$X \backslash Y$	4	5	6	7	Total
1	0.01	0.05	0.1	0.12	0.28
2	0.12	0.2	0.01	0.07	0.4
3	0.05	0.09	0.05	0.13	0.32
Total	0.18	0.34	0.16	0.32	1

- (iii) A) If discrete random variable X has Uniform Distribution over the range $\{0, 1, 2, \dots, n\}$. Its variance is 33.25. Find n and its mean.
 B) A taxi cab company has 10 Ambassadors and remaining 5 cars are of other. A person wants to hire 7 taxis for a marriage party by random choice. Find the probability that he chooses
 (a) All Ambassador (b) 3 Ambassadors than remaining.