

- extra*
- Note: 1) All Questions are compulsory.
 2) All Questions have equal marks
 3) Figures to right indicate full marks of questions
 4) Use of non-programmable scientific calculator is allowed.

Q.1. A) State whether the following statements are true or false .If true justify the statement .If false correct it. (10)

- 1) If $P(A/B) = P(A)$ then A depends on B
- 2) $P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$
- 3) If X and Y are two random variables with $V(X) = 2$ and $V(Y) = 3$
 $V(2X + 3Y) = 35$
- 4) For a discrete random Variables $XE(X) > \sqrt{E(x^2)}$
- 5) The binomial distribution is symmetric when $p > q$

B) Answer in One sentence

- 1) Define Binomial distribution
- 2) State multiplication theorem expectation.
- 3) If cumulative Distribution functions of discrete random variable X is

$$\begin{aligned} F(x) &= 0 & x < 0 \\ &= \frac{1}{6} & 0 \leq x < 1 \\ &= \frac{1}{4} & 1 \leq x < 2 \\ &= \frac{7}{8} & 2 \leq x < 3 \\ &= 1 & x \geq 3 \end{aligned}$$

Find $P(1.3 < x \leq 2.8)$

- 4) Give statistical definition of probability
 - 5) If A and B are independent events then show that $P(A \cup B) = P(A) + P(\bar{A})P(B)$
- Q.2. Attempt any two sub questions (10 marks each question) (20)
- 1) A) State and prove that BAYES Theorem.
 B) A man has 5 one rupees coins and one of them is known to have two heads. He select one coin at random and tosses it 5 times. If it always falls head upwards, what is the probability that it is the coin with two heads?
 - 2) State and prove multiplication theorem on probability hence find $P(A), P(A \cap B), P(A/B)$ and $P(A \cup B)$ if $P(\bar{A}) = \frac{3}{4}$ and $P(B) = \frac{1}{5}$.
 - 3) i) Give any mathematical definition of a probability and state its limitations
 ii) Define the following
 - a) Deterministic experiments
 - b) Objective Probability
 - c) Complimentary Event
 - d) Mutually Exclusive Event

e) Exhaustive event

4) A committee of four teachers is to be formed among 3 teachers of Arts faculty, 4 of Science faculty, 2 of Commerce faculty and 1 from self-financing courses. Find the probability that i) All the 4 faculties are represented in the committee ii) The committee will have the teacher from self-financing course and at least 1 teacher from Arts faculty. iii) The committee will have no teacher from self-financing and exactly 1 teacher from commerce faculty.

Q.3. Attempt any two sub questions (10 marks each question)

(20)

1. i) Define mathematical expectation Discrete random variables X
ii) Show that $\epsilon(x-a)^2$ is minimum when $a = \epsilon(x)$
iii) State the effect of change of origin and scale of expectation.
2. i) Define a) co-variance between X and Y. b) Correlation coefficient between X and Y. Further state what are they if X, Y are independent?
ii) With suitable illustration explain the following for discrete random variable X and Y
a) Joint probability mass function of X and Y
b) Marginal probability mass function of X and Y
c) Conditional probability mass function of Y given $X = x$
3. i) State and prove that the additional theorem on expectation for two random variable.
ii) Show that $\text{COV}(X, Y) = E(XY) - E(X)E(Y)$
iii) X and Y are two stochastically independent random variables with means 10 & 12 and 9 & 16 respectively compute (a) $E(7x-6y+4)$ (b) $V(4x-3y+2)$ (c) $E(XY)$
4. A fair coin is tossed 4 times let X denotes the number of heads occurring in the last two tosses and Y denote the total number of heads occurring in the four tosses. Find the joint probability distribution of X and Y. Also obtain the conditional probability distribution of Y given $X=1$ also find $E(XY)$

Q.4. Attempt any two sub questions (10 marks each question)

(20)

1. Define hyper geometric distribution derive its mean and variance
2. i) Describe important features of binominal distribution. Also find recurrence relation for the probabilities of binominal distribution.
ii) An unbiased coin is tossed five times. What is the probability of getting at least two heads?
3. Define Poisson Distribution derived its variance also important features of Poisson distribution
4. Define discrete uniform distribution hence solve random variable X has probability mass function given by $P(x) = \frac{1}{K+1}, X=0,1,2,\dots, K$
 $=0$ otherwise

Q.5. Attempt any four of the following (Five marks each question)

(20)

1. Derive Mean and variance of Bernoulli Distribution
2. For Poisson approximation to Binominal distribution prove that
$$\lim_{n \rightarrow \infty} [n_c x P^x (1-P)^{n-x}] = \frac{e^{-m} m^x}{x!} \text{ where } m = np$$
3. A discrete variable X has the following probability distribution

X	1	2	3	4
P(X)	0.4	0.3	0.2	0.1

Obtain the first four row moments about origin. Calculate coefficient of skewness and kurtosis and comment.

4. A Random variable X takes values 1,3,5 with probabilities $\frac{3}{5}$, $\frac{3}{10}$ and $\frac{1}{10}$ respectively. Cumulative function, $F(x)$ of X hence obtain $F(1-3)$, $F(6)$ also draw the graph of cumulative distribution function
5. If A and B are independent events then prove that i) \bar{A} and B ii) A and \bar{B} and iii) \bar{A} and \bar{B} . A and B are independent.
6. Two fair dice are rolled simultaneously. Write the sample space of the experiment Find the probability that the sum of the number of the upper most faces is (i) 8, (ii) less than 5, (iii) multiple of 4
7. The probability mass function of a discrete random variable X is given by

X	-2	-1	0	1	2
$P(x)$	0.1	0.15	0.2	0.15	0.4

Find $P(x \geq -1)$ Obtain the probability distribution of $y = x^2$