Q.P. Code: 51728

(2 $^{1}/_{2}$ Hours) **[Total Marks: 75]**

N. B. : (1) All questions are compulsory.
(2) Figures to the right indicate full marks.
(3) Draw neat diagrams wherever necessary.
(4) Symbols have usual meaning unless otherwise stated.
(5) Use of log table and non-programmable calculator is allowed.

1. (a) Attempt any one:---
   (i) Set up the equation of a particle in an inverse square field. Solve it and state the conditions under which the path of the particle will be an ellipse, a parabola or a hyperbola. 10
   (ii) A starred system S* rotate with a variable angular velocity $\overrightarrow{\omega}$ with respect to an inertial system S fixed in space. Show that 10
   $$\frac{d\vec{r}}{dt} = \frac{d^*\vec{r}}{dt} + \overrightarrow{\omega} \times \vec{r}.$$
   Hence obtain the Coriolis theorem.

   (b) Attempt any one:---
   (i) Show that when a body moves in a central force field its aerial velocity is constant. 5
   (ii) Calculate the time taken by the plane of oscillations of a pendulum at 30° latitude to turn through a right angle. 5

2. (a) Attempt any one:---
   (i) Derive Lagrange's equation of motion in several dimensions with no constraint imposed. 10
   (ii) Explain how the forces of constraints are determined and used in Lagrangian formulation. Illustrate the same taking Atwood's machine as an example 10

   (b) Attempt any one:---
   (i) Describe generalized coordinates. 5
   (ii) For a system of N particles show that kinetic energy has a form 5
   $$T = T_2 + T_1 + T_0$$
   Where T_2 contains quadratic terms in generalised velocity, T_1 contains linear term and T_0 is independent of velocities. .

3. (a) Attempt any one:---
   (i) What is an ideal fluid? Obtain the Euler's equation of motion for an ideal fluid. State the assumptions. 10
   (ii) Derive Euler's equation of motion for a rigid body. Solve these equations for torque free rotational motion of symmetric body and hence show that the magnitude of angular velocity vector is a constant. 10

   (b) Attempt any one:---
   (i) If no external force acts on the fluid element and the pressure is constant throughout the medium, show that the angular momentum of the fluid element is constant. 5

(ii) The Lagrangian for a symmetric top is 5

$$L = \frac{1}{2}I_1\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right) + \frac{1}{2}I_3\left(\dot{\psi} + \dot{\phi}\cos\theta\right)^2 - mgl\cos\theta$$

Obtain Lagrange's equation for Euler's angles θ and ψ.

4. (a) Attempt any one:---

(i) State Duffing's equation for a driven damped anharmonic oscillator. Discuss the features of the numerical solution of the Duffing's equation for the two cases, 10

1) $\gamma = 0.1$ and $f = 0.5$. (2) $\gamma = 0.1$ and $f = 3$.

(ii) What is logistic map? Using quadratic map obtain the equation for the slope of the tangent drawn at a fixed point and hence explain the stability of the fixed points for 1) $0 < \lambda < 1$ 2) $1 < \lambda < 3$ 3) $3 < \lambda < 4$. 10

(b) Attempt any one:---

(i) Draw and explain phase space diagram for undamped, damped and driven oscillator. 5

(ii) Find the value of d, the fractal dimension of Sierpinski Gasket. Draw the diagrams. 5

5. (a) Attempt any one:---

(i) If the eccentricity of a planet's orbit about the sun is 0.4, find the ratio of the lengths of the axis to the minor axes of the orbit of the planet. 4

(ii) A body of mass 1 kg is falling freely under gravity. Find the fictitious force and the total force acting on the body as observed from a frame moving vertically downward with an acceleration of 2 m/s^2. 4

5. (b) Attempt any one:---

(i) For a particle moving in a plane, obtain the Lagrange's equation of motion in polar coordinates. 4

(ii) Body is moving freely in space (no force acting on body). Write down its Lagrangian function and mention the cyclic coordinates considering Cartesian axes frame, hence find constant of motion. 4

5. (c) Attempt any one:---

(i) Velocity of a fluid is given by $\vec{v} = (bxt)\hat{i}$. Find acceleration $\vec{a}(x,t)$ of the fluid element at position x and time t. 4

(ii) A rigid body consists of three particles of masses 2, 1 and 4 units located at $(1,-1,1)$, $(2,0,2)$ and $(-1,1,0)$ respectively. Determine the elements of the moment of inertia matrix for the rigid body. 4

5. (d) Attempt any one:---

(i) Find fractal dimension of Cantor Set. 3

(ii) Draw the logistics maps for $\lambda = 0.8$. Find the corresponding fixed points. 3
