**Q.P. Code: 51731**

(**$2\frac{1}{2}$ Hours**) **[Total Marks: 75]**

N. B. : (1) All questions are compulsory.
(2) Figures to the right indicate full marks.
(3) Draw neat diagrams wherever necessary.
(4) Symbols have usual meaning unless otherwise stated.
(5) Use of log table and non-programmable calculator is allowed.

1. (a) Attempt any one:---
   (i) Explain the term "effective gravitational acceleration". Why is it minimum at the equator and maximum at the poles? Explain how the "effective gravitational acceleration" explains the shape of the earth. 10
   (ii) Define and explain with suitable diagram the following terms: Angle of scattering, Impact parameter, Differential scattering cross section. 10

   (b) Attempt any one:---
   (i) Show that the centrifugal acceleration contribution to $g_e$ (effective gravitational acceleration) is less than 0.3% of g. (Given: $R_E = 6378$ km) 5
   (ii) A comet has a period of revolution T = 36 years around the sun. Determine the semi major axis of its orbit in AU. 5
   ( 1 AU = $1.5 \times 10^8$ km ( sun to earth distance) )
   G = $6.67\times 10^{-11}$ $Nm^2/kg^2$ , Mass of the sun = $2.0 \times 10^{30}$ kg

2. (a) Attempt any one:---
   (i) State the D'Alembert's principle and derive an expression for Lagrange's equation with one degree of freedom. 10
   (ii) Explain how the forces of constraints are determined and used in Lagrangian Formulation. Illustrate the same taking Atwood's machine as an example. 10

   (b) Attempt any one:---
   (i) What are generalized co-ordinates? Explain the statement "More the constraints imposed on the system lesser the number of generalized coordinates describing the system" 5
   (ii) What are constants of motion and ignorable coordinates? Explain in brief. 5

3. (a) Attempt any one:---
   (i) In the case of a liquid, derive the conservation equation for energy in the form of Bernoulli's theorem. 10
   (ii) What is symmetric body? Show that when no external torque is applied to a freely rotating symmetric body, the angular velocity of the body precesses describing a cone about the axis of symmetry of the body. 10

   (b) Attempt any one:---
   (i) Explain the term: streamline flow, steady flow and tubes of flow. 5

   (ii) When a torque $\vec{N}$ acts on the rigid body rotating about fixed point then show that the rate of change of kinetic energy is 5
   $$\frac{dT}{dt} = \vec{\omega} \cdot \vec{N}$$

4. (a) Attempt any one:---

(i) The potential energy of a one dimensional damped anharmonic oscillator is given by $V(x) = K\left(\frac{x^2}{2} + \frac{\alpha x^4}{4}\right)$, where K is the spring constant and α is anharmonic coefficient. Discuss the potential energy curve for various combinations of K and α. Comment on confinement of motion. 10

(ii) What is logistic map? Using quadratic map obtain the equation for the slope of the tangent drawn at a fixed point and hence explain the stability of the fixed points for 1) $0 < \lambda < 1$ 2) $1 < \lambda < 3$ 3) $3 < \lambda < 4$ 10

(b) Attempt any one:---

(i) State Duffings equation for a driven damped anharmonic oscillator. Discuss the features of the numerical solution of the Duffing's equation for 1) $\gamma = 0.1$ and $f = 0.5$ 5

(ii) Calculate the fractal dimensions of (1) A line of length $\ell$ in a plane. (2) A square of area $\ell^2$. 5

5. (a) Attempt any one:---

(i) Find the latitude of a place where the plane of vibration of a pendulum rotates once a day. 4

(ii) If the eccentricity of a planet's orbit about the sun is 0.8 and the semi major axis of planet's orbit is 15 AU, then find minimum and maximum distances of the planet from the centre of the sun in AU. 4

5. (b) Attempt any one:---

(i) For a particle moving on surface of a sphere of radius r. Set up its equation of Motion in spherical polar coordinates using Lagrangian Formalism. 4

(ii) Set up Lagrangian for a simple pendulum and obtain equation describing its motion. 4

5. (c) Attempt any one:---

(i) When is the flow of a liquid said to be irrotational? If the velocity of a liquid elements at a point is $\vec{v} = x^2\hat{i} + 3yz\hat{j}$, is the flow irrotational? 4

(ii) The moment of inertia tensor of a rigid body is 4

$$II = Ma^2\begin{pmatrix} 1 & -3 & 1 \\ -3 & 2 & 5 \\ 1 & 5 & 2 \end{pmatrix}$$

The angular velocity of the body is

$$\vec{\omega} = 3\hat{i} - 2\hat{j} - 6\hat{k}$$

Compute the angular momentum and the kinetic energy of the body.

5. (d) Attempt any one:---

(i) Calculate the fixed point for the quadratic maps drawn for $\lambda = 0.8$ and $\lambda = 2.0$. 3

(ii) Find Hausdorf dimension of Cantor Set. 3

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