

(3 Hours)

[Total Marks 100]

N. B.:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Draw neat diagrams wherever necessary.
4. Symbols have usual meaning unless otherwise stated.
5. Use of non-programmable calculator is allowed.

Q 1. Attempt any **two**:

- (i) State and prove Kepler's laws of planetary motion. 10
- (ii) Obtain the equation of motion of a particle of mass 'm' as related to the rotating earth. Hence show that the angle between g and g_e depends on colatitude angle Θ . 10
- (iii) Derive an expression for Coriolis theorem. Interpret each term. 10

Q 2. Attempt any **two**:

- (i) What is meant by generalized co-ordinates? Derive an expression for generalized velocity and generalized kinetic energy. 10
- (ii) Obtain Lagrange's equation of motion by using D'Alembert's principle for holonomic systems. 10
- (iii) A double pendulum consists of two weightless rods connected to each other and a point of support. The masses m_1 and m_2 are not equal but the length of the rods are equal. Pendulums are free to swing only in one vertical plane. Derive the Lagrangian for the system. 10

Q 3. Attempt any **two**:

- (i) For fluids, derive the conservation equation for energy in the form of Bernoulli's theorem. 10
- (ii) Derive Euler's equations of motion of a rigid body. Solve these equations for a torque free rotational motion of symmetric body and hence show that the magnitude of the angular velocity vector is a constant. 10
- (iii) A rigid body is constrained to move about a point which is fixed. Derive an expression for its angular momentum about the instantaneous axis of rotation passing through the fixed point. Hence derive an expression for kinetic energy, $T = \frac{1}{2} \vec{\omega} \cdot \vec{L}$. 10

Q 4. Attempt any **two**:

- (i) The potential energy of a one dimensional anharmonic oscillator is given by $V(x) = K \left(\frac{x^2}{2} + \frac{\alpha x^4}{4} \right)$, where K is the spring constant and α is anharmonic coefficient. Discuss the potential energy curve for various combinations of K and α . 10
- (ii) Discuss fixed points of a logistic map, stability of fixed points and periodic attractors. Explain how series of bifurcations lead to chaos when $\lambda = 3.57$. 10
- (iii) Discuss numerical solutions of Duffing's equation for $\gamma = 0.1$, and $f = 0.5$ & $f = 3$ and also explain why the two solutions for even harmonics are different. 10

Q 5. Attempt any **four**:

- (i) Show that when a body moves in a central force field its motion is confined to a plane. 5
- (ii) Find the Coriolis force acting on a body of mass 1kg moving northward with a horizontal velocity of 100 m/s at a place on earth whose colatitudes is 60° N 5
- (iii) Define constraints. With good examples, explain holonomic and non-holonomic constraints. 5
- (iv) Define cyclic coordinate. Which coordinate is called as cyclic coordinate in equation given below 5
 - a) $L = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) - mgy$
 - b) $p_\theta = mr^2\dot{\theta}$
- (v) Consider a fluid flow in which velocity is given by, $\vec{v}(x, t) = \frac{at}{x} \hat{i}$, $x > 0$. 5
Find the acceleration $\vec{a}(x, t)$ of the fluid element at position x and time t .
- (vi) If a rigid body consists of three particles of masses 2, 1 and 4 grams located at $(+1, -1, 1)$, $(2, 0, 2)$ $(-1, 1, 0)$ cm respectively. Find principal moment of inertia and product of moment of inertia. 5
- (vii) Two very close initial values of x on logistic map are 0.50000 and 0.50002 respectively. With $\lambda = 4$ after 20 iterations the values are 0.08561 and 0.00561 respectively. Calculate Lyapunov exponent. 5
- (viii) What is phase space diagram? Plot the phase space diagram for one dimensional oscillator. 5