Q.P. Code: 51729

(2 1/2 Hours) **[Total Marks: 75]**

N. B. : (1) All questions are compulsory.
(2) Figures to the right indicate full marks.
(3) Draw neat diagrams wherever necessary.
(4) Symbols have usual meaning unless otherwise stated.
(5) Use of log table and non-programmable calculator is allowed.

1. (a) Attempt any one:---
   (i) Derive the equation of motion of a particle of mass 'm' under the action of central force and reduce the problem to one dimensional problem using the effective potential energy of the particle. 10
   (ii) A starred coordinate system rotates relative to an unstarred system fixed in space. Both systems have common origin. Prove that 10

$$\frac{d^2\vec{A}}{dt^2} = \frac{d^{*2}\vec{A}}{dt^2} + \vec{\omega} \times (\vec{\omega} \times \vec{A}) + 2\vec{\omega} \times \frac{d^*\vec{A}}{dt} + \frac{d^*\vec{\omega}}{dt} \times \vec{A}$$

   Interpret the various terms in the equation.

   (b) Attempt any one:---
   (i) State Kepler's laws of planetary motion. Show that each planet describes an ellipse with the sun at one focus 5
   (ii) A body is dropped from rest from a height of 50 m. Calculate the deviation from the vertical suffered by the body, due to Coriolis force, when it reaches the surface of the earth. 5
   Latitude of place = 60° N, g = 9.8 m/s^2

2. (a) Attempt any one:---
   (i) State the D'Alembert's principle and derive an expression for Lagrange's equation with one degree of freedom. 10
   (ii) What are ignorable coordinates? Show that if the Lagrangian function of a system does not depend explicitely on time the total energy of the system is conserved. 10

   (b) Attempt any one:---
   (i) What are generalised co-ordinates? Explain the statement 'More the constraints imposed on the system lesser the number of generalized coordinates describing the system. 5
   (ii) Show that momentum conjugate to a cyclic coordinate is conserved. Give an example. 5

3. (a) Attempt any one:---
   (i) Show $\frac{d}{dt}(\delta V) = (\nabla \cdot \vec{v})\delta V$ for a fluid element. Hence derive the equation of continuity for the motion of continuous matter. 10

(ii) A symmetric top spinning about the axis of symmetry inclined to the vertical by an angle θ has its lower tip fixed. Its K.E. is given by: 10

$$T = \frac{1}{2} I_1 \left(\dot{\theta}^2 + \dot{\phi}^2 sin^2\theta\right) + \frac{1}{2} I_3 \left(\dot{\psi} + \dot{\phi} \cos\theta\right)^2$$

Show that the top will preccess without nutations if the spin $\omega_3 \geq \frac{2}{I_3}\sqrt{mglI_1 \cos\theta_0}$.

(b) Attempt any one:---

(i) If the total external force includes the body force and external force on volume V due to the pressure exerted by surrounding fluid on it and the total external force acting on the fluid volume V of the fluid is zero, show that its linear momentum is constant. 5

(ii) Show that the kinetic energy of a rigid body is given by $T = \frac{1}{2}\vec{\omega} \cdot \vec{L}$ 5

4. (a) Attempt any one:---

(i) The potential energy of a one dimensional damped anharmonic oscillator is given by $V(x) = K\left(\frac{x^2}{2} + \frac{\alpha x^4}{4}\right)$, where K is the spring constant and α is anharmonic coefficient. Discuss the potential energy curve for various combinations of K and α. Comment on confinement of motion. 10

(ii) State Duffing's equation for a driven damped anharmonic oscillator. Discuss the features of the numerical solution of the Duffing's equation for the two cases 10

1) $\gamma = 0.1$ and $f = 0.5$ 2) $\gamma = 0.1$ and $f = 3$.

(b) Attempt any one:---

(i) Find Hausdorf dimension of Cantor Set. 5

(ii) Show that in Logistic map for $\lambda = 0.6$, the fixed point $x = 0$ is an attractor. 5

5. (a) Attempt any one:---

(i) If a body of mass 500 gm is moving with a velocity of 200 m/s, estimate the maximum Coriolis force experienced by the body. 4

(ii) Halley's comet has a period of revolution T = 76 years around the sun. Determine the semi major axis of its orbit in AU. 4

(1 AU = 1.5×10^8 km (sun to earth distance))

G = 6.67×10^{-11} Nm^2/kg^2 , Mass of the sun = 2.0×10^{30} kg

5. (b) Attempt any one:---

(i) For a particle moving in a plane, write down the Lagrange's equation of motion in polar coordinates. 4

(ii) Set up Lagrangian for a simple pendulum and obtain equation describing its motion. 4

5. (c) Attempt any one:---

(i) Water is flowing out of a pipe of non-uniform cross section at a steady rate. The area of cross section at the two points A and B of the tube are $0.0314m^2$ and $0.0628m^2$ respectively. The pressure difference between A and B is $9800N/m^2$. Density of water is $1000kg/m^3$. Find the rate of flow of water through the pipe. 4

(ii) OX, OY and OZ are the principal axes of a rigid body, at fixed point O. If $I_x = I_y$ and $\vec{\omega}$ is in XY plane, show that the angular momentum about O is parallel to $\vec{\omega}$. 4

5. (d) Attempt any one:---

(i) Calculate the fixed point for the quadratic maps drawn for $\lambda = 0.3$ and $\lambda = 2.4$. 3

(ii) With the help of Butterfly effect define chaos. 3
