

N.B. : (1) All questions are compulsory.

(2) **Figures** to the **right** indicate **full** marks.

(3) Draw **neat** diagrams wherever **necessary**.

(4) Symbols have usual meaning unless otherwise stated.

(5) Use of **log-table** and **non-programmable** calculator is allowed.

1. (a) Attempt any **one**:---

- (i) Consider two coordinate systems S^* and S such that they have the common origin. S^* is rotating with angular velocity $\vec{\omega}$ with respect to S . Show that $\frac{d\vec{r}}{dt} = \frac{d^*\vec{r}}{dt} + \vec{\omega} \times \vec{r}$. Hence obtain the Coriolis theorem. 10
Name the terms in the theorem.

- (ii) Set up the equation of motion of a particle moving in an inverse square field. Show that the solution is equation of conic section. Hence derive the condition that the path of the particle will be an ellipse. 10

(b) Attempt any **one**:---

- (i) Describe scattering of alpha particles. Hence obtain equations for the differential cross-section and the impact parameter. 5
(ii) State and explain Larmor's Theorem. Discuss its possible application. 5

2. (a) Attempt any **one**:---

- (i) Describe the D'Alembert's principle in detail and derive Lagrange's equations using this principle. 10
(ii) A mass 'm' moves in a vertical plane subject to gravitational force and to the tension force of an attached string of length ℓ . The other end of the string is held fixed in position O. The constraint on the system is that the mass always has to be at distance ℓ from the Fixed centre O. 10
a) Derive the equation of the mass.
b) Find the force of constraint.

(b) Attempt any **one**:---

- (i) A body is moving freely in space (no force acts on it). Write down its Lagrangian function and mention the cyclic co-ordinates considering a Cartesian axes frame. Hence find the constant of motion. 5
(ii) Derive an expression for the kinetic energy of a system of particles in generalized coordinates. 5

3. (a) Attempt any **one**:---

- (i) Derive the equation of continuity for a fluid moving with a velocity 'v' and density ' ρ '. 10
(ii) Derive Euler's equations of motion of a rigid body. Solve these equations for torque free rotational motion of a symmetric body and hence show that the magnitude of the angular velocity vector is a constant. 10

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- (b) Attempt any **one**:---
- Explain the terms: streamline flow and steady flow. 5
 - Explain the rotation of the earth as a free symmetric top. 5
4. (a) Attempt any **one**:---
- What is a logistic map? Calculate the fixed points for $\lambda = 0.7$ and $\lambda = 1.4$. 10
Starting with $x_0 = 0.9$, tabulate the iterations up to x_{10} for these values of λ and argue that these fixed points are stable attractors explaining what is meant by it. Draw necessary graphs.
 - What is a fractal? Explain the construction of a Cantor set and find its dimension. Also explain its emergence in the attractor of the Henon map. 10
- (b) Attempt any **one**:---
- Graphically illustrate potential energy of an anharmonic oscillator constructed using a hard spring and subjected to a conservative force $F(x) = -k(x + \alpha x^3)$, for $k = \pm 1$. 5
 - State Duffing's equation. With the help of a necessary graph, explain the mechanical hysteresis. 5
5. (a) Attempt any **one**:---
- Jupiter orbits the Sun at 5.20 AU. Find its period in years and days. 4
($G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$, $1 \text{ AU} = 1.5 \times 10^8 \text{ km}$ and $\text{Mass of the Sun} = 2.0 \times 10^{30} \text{ kg}$)
 - Find the horizontal component of the Coriolis force on a body of mass 2 kg moving in northward direction with speed 10 m/s at Mumbai (latitude 20°N). 4
- (b) Attempt any **one**:---
- Prove that when a coordinate q_j does not occur explicitly in the Lagrangian function, the corresponding momentum p_j is constant. 4
 - Derive the following relation: 4
$$p = p_c \frac{\partial x}{\partial q}$$
Where p and p_c are the general and Cartesian momentum respectively.
- (c) Attempt any **one**:---
- Consider a fluid flow given by $\mathbf{v} = by\mathbf{i}$ in a coordinate system s . Is the fluid incompressible? Explain. 4
 - Three point masses of 'm' each are placed at the three corners O, A, and B of the triangle. Coordinates of O, A and B are (0,0,0), (2,2,0) and (2,-2,0) in a coordinate frame (x,y,z). Show that if the system behaves as a rigid body, the axes ox, oy and oz are the principal axes of the body. 4
- (d) Attempt any **one**:---
- The symmetric tent map is defined by a function $f(x) = 2x$ when $0 \leq x \leq 0.5$ and $f(x) = 2 - 2x$ when $0.5 \leq x \leq 1$. Sketch this function and find the fixed points. 3
 - In a variation of Cantor set, a line segment is divided into five equal segments and the middle one is removed. Then this process is continued on each of the remaining four segment. Find the fractal dimension of the resulting set. 3