

(3 hrs)

Total Marks : 100

N.B. : (1) All questions are compulsory.

(2) Figures to the right indicate full marks.

(3) Draw neat diagrams wherever necessary.

(5) Symbols have usual meaning unless otherwise stated.

(5) Use of non-programmable calculator is allowed.

Constants : $h = 6.626 \times 10^{-34}$ Js, $k = 1.38 \times 10^{-23}$ JK⁻¹

1. Attempt any two:---

(a) State and explain probability theorems. Using this theorem, find the chance of throwing a 6 at least once in two throws of a single die. **10**

(b) Explain the terms **10**

i) Binomial probability functions

ii) Chebyshev's inequality

iii) Laws of large number

(c) Explain the Poisson distribution and derive the required relations. **10**

2. Attempt any two:---

(a) State the second order non-homogeneous linear ordinary differential equation with constant variable and solve the same. **10**

(b) What do you mean by partial differential equation? Hence Solve the equation **10**

$$\frac{\partial^2 z(x,y)}{\partial x \partial y} = x^2 y$$

For $z(x, 0) = x^2$ and $z(1, y) = \cos(y)$

(c) What do you mean by Hyperbolic function in complex number. **10**

Prove the following formulas.

i) $\cosh^2(z) - \sinh^2(z) = 1$

ii) $\frac{d}{dz} \cosh(z) = \sinh(z)$

3. Attempt any two:---

(a) Derive an expression of root mean square deviation in occupation number of dominate configuration. **10**

(b) State Boltzmann formula of entropy and derive its relation with Canonical partition function **10**

(c) Define partition function. Derive expression for translation partition function. Find translation partition function for Ar confine to a volume of 1L at 298K. (Given $m_{Ar} = 6.63 \times 10^{-26}$ kg) **10**

4. Attempt any two:---
- (a) Consider a large box having area A is divided into k cells of area $a_1, a_2, a_3, \dots, a_k$. N identical balls are thrown in the box in a completely random manner. Hence find the condition for the most probable distribution of N balls. **10**
- (b) What are Fermions? Derive Fermi-Dirac distribution law. **10**
- (c) Derive Rayleigh-Jean's formula for the black body radiation. **10**
5. Attempt any **Four**:---
- (i) A club consists of 50 members. In how many ways can a president, vice-president, secretary and treasurer be chosen? In how many ways can a committee of 4 members be chosen? **05**
- (ii) Three coins tossed, the number of random variables x (say head) are 0, 1, 2 & 3. Calculate the followings **05**
- a) Mean value of x .
- b) Variance of x .
- c) Standard deviation of x .
- (iii) Find the value of $\sin\left(\frac{\pi}{2} + i \ln 3\right)$ **05**
- (iv) Solve $y'' - 2y' - 3y = e^{2x}$ **05**
- (v) Write notes on Equipartition theorem **05**
- (vi) What is the difference in energy between the $n = 2$ and $n = 1$ states for molecular oxygen constrained by a one-dimensional box having a length of 1.00 cm? (Given mass of $O_2 = 5.31 \times 10^{-25}$ kg.) **05**
- (vii) If the r.m.s. velocity of the molecules of hydrogen at N.T.P. is 1.84 km/s, calculate the r.m.s. velocity of oxygen molecules at N.T.P. Molecular weight of hydrogen and oxygen are 2 and 32 respectively. **05**
- (viii) Three identical particles can be in any of the five states. What are the number of possible ways of distributing them in various states according to **05**
- (a) M-B. Statistics. (b) B.E. Statistics. (c) F.D. Statistics.
