

QP Code : 19398

[Time : 2½ Hours

[Total marks : 75

- N.B :**
1. All questions are **compulsory**.
 2. **Figures** to the **right** indicate **full** marks.
 3. Draw **neat** diagrams wherever **necessary**.
 4. Symbols have usual meaning unless otherwise stated.
 5. Use of **non-programmable** calculator is allowed.

Given: Boltzman constant: $k = 1.38 \times 10^{-23}$ J/K, $1\text{eV} = 1.6 \times 10^{-19}$ J

Table of Laplace Transforms

	$y=f(t)$	$Y=L(y) = F(p)$	
L2	e^{-at}	$\frac{1}{p+a}$	$R_e(p+a) > 0$
L13	$e^{-at} \sin bt$	$\frac{b}{(p+a)^2 + b^2}$	$R_e(p+a) > lm b $
L14	$e^{-at} \cos bt$	$\frac{p+a}{(p+a)^2 + b^2}$	$R_e(p+a) > lm b $

1. (a) Attempt any **one** :—
 - (i) Explain the Poisson distribution and derive the required relations. **10**
 - (ii) Write a detailed summary of Methods of counting. **10**
 (b) Attempt any **one** :—
 - (i) State and explain the laws of Large Numbers. **5**
 - (ii) What do you understand by probability functions ? **5**
2. (a) Attempt any **one** :—
 - (i) Explain the method of solving the differential equations by Laplace Transform and solve the following equation: $y'' + 4y' + 13y = 20e^{-t}$, where $y_0 = 1$ and $y'_0 = 3$. **10**
 - (ii) Describe the method of solving the second order non-homogenous linear ordinary differential equations with constant coefficients. Solve the following equation: $y'' + 2y' + y = x$. **10**
 (b) Attempt any **one** :—
 - (i) Solve the equation $(D^2 + 5D + 4)y = \cos 2x$, where $D = \frac{d}{dx}$. **5**
 - (ii) Find the Fourier Transform of $f(x) = \begin{cases} 1, & |x| \leq a \\ 0, & |x| \geq a \end{cases}$. **5**
3. (a) Attempt any **one** :—
 - (i) Consider two interactive systems A and A' . They are neither adiabatically isolated nor their external parameters are kept fixed. Show that in equilibrium the temperature and pressure on both sides of the partition become equal. **10**

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- (ii) Describe the concept of Gibb's free energy (G). Use this concept to explain co-existence of two phases and the phase equilibrium curve. Hence discuss the triple point. 10
- (b) Attempt any **one** :—
- (i) Show that the entropy of a system in a heat bath is given by : 5
- $$S = -k \sum_r P_r \ln P_r$$
- (ii) Define microstates, macrostate and system state. Two states of a system have an energy difference of 0.028 eV. The relative probability that the lower state be occupied with respect to the upper state is e^3 . Calculate temperature of the system. 5
4. (a) Attempt any **one** :—
- (i) Consider a large box having area A is divided into k cells of area a_1, a_2, \dots, a_k . N identical balls are thrown in the box in a completely random manner. Obtain the condition for the most probable distribution of N balls in k cells. 10
- (ii) What are Bosons? Derive Bose-Einstein distribution law. 10
- (b) Attempt any **one** :—
- (i) Show that, at low frequency Planck's radiation formula reduces to Rayleigh-Jeans formula. 5
- (ii) If the mean velocity of hydrogen molecules at a given temperature is 1.45 km/sec, calculate their root mean square velocity and most probable velocity at the same temperature. 5
5. (a) Attempt any **one** :—
- (i) Two students are working separately on the same problem. If the first student has probability 1/2 of solving it and the second student has probability 3/4 of solving it, what is the probability that at least one of them solves it? 4
- (ii) If 1500 people each select a number at random between 1 and 500, what is the probability that 2 people selected the number 29? 4
- (b) Attempt any **one** :—
- (i) Show that the cosine transform of the first order derivative is given by 4
- $$F_c^{(1)}(k) = k F_s(k) - \sqrt{\frac{2}{\pi}} f(0)$$
- (ii) Calculate the Laplace Transform $F(p)$ for $f(t) = 1$ and $f(t) = \cos at$. 4
- (c) Attempt any **one** :—
- (i) Consider a system of a single particle (with zero spin) inside a box. If energy of the particle in the ground state is E, find energy and degeneracy of the particle in its fifth excited state. 4

- (ii) A system is having 40 spin half particles fixed in space and placed in magnetic field **B**. Find relative probability of 10% deviation $[R_{(10\% \text{ off})}]$ from the most probable macrostate. 4
- (d) Attempt any **one** :—
- (i) Consider distribution of 7 electrons in 10 energy cells. Find the number of possible arrangements using : (1) B-E statistics, (2) F-D statistics. Comment on the results if any. 3
- (ii) Solar spectrum shows that, maximum energy is emitted at wavelength 5500 Å. Assuming the sun as a black body, estimate its temperature. 3
(Given Wien's Constant = 0.29 cmK)

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