T.Y.BSC
Sem V

Subject: Physics PI
Mathematical and Statistical Physics. 2016-17

(5)

QP Code : 77032

(2½ Hours) **[Total Marks : 75**

N. B. : (1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicate **full** murks.
(3) Use of non-programmable calculators and log-tables is allowed.
(4) Draw neat diagrams wherever necessary.

1. (A) Attempt any **one** :–

(i) Discuss the method of solving the second order homogeneous linear ordinary differential equations with the constant coefficients. **10**

(ii) (a) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^x$. **10**

(b) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 2\cos x$.

(B) Attempt any **one** :–

(i) Solve $\frac{\partial^2 z(x,y)}{\partial x\, \partial y} = x^2 y$ **5**

Subject to the condition $z(x, 0) = x^2$ and $z(1, y) = \cos y$

(ii) Test the following equation for exactness and find its solution. **5**
$x(x^2 + 2y^2)\, dx + y(2x^2 + y^2)\, dy = 0$

2. (A) Attempt any **one** :–

(i) State cosine and sine transform pairs. Find the fourier sine and cosine transform of a function : $f(x) = e^{-ax}$ **10**

(ii) Expand $f(x) = x$ in the interval $-\pi \le x \le \pi$ in a Fourier series and show that **10**

$$f(x) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin nx}{n}$$

Also show the graphical representation in the interval and outside the interval.

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KS-Con. 3449-16.

(B) Attempt any **one** :– 5

(i) Find the Fourier transform of the box function f(x), where

$$f(x) = \begin{cases} 1 & \text{for} \quad -a \le x \le a \\ 0 & \text{for} \quad |x| > a \end{cases}$$

(ii) Obtain complex form of Fourier series. Hence show that the coefficient is given by 5

$$Cn = \frac{1}{2l}\int_{-l}^{l} f(x)\, e^{-in\pi x/l}\, dx$$

3. (A) Attempt any **one** :– **10**

(i) Consider two interactive systems A and A'. They are neither adiabatically isolated nor their external parameters are kept fixed. Show that, at equilibrium the temperature and pressure on both sides of the partition became equal.

(ii) Describe the concept of number of states accessible to a system. Consider a system of single particle in a cubical box. Find the number of particle states between energy E and E + dE. **10**

(B) Attempt any **one** :–

(i) Consider a system in heat bath. Use concept of ensemble averge to show that the entropy of the system is : $S = -k \sum P_r \ln P_r$. **5**

(ii) Describe the concept of Helmolthz free energy of a system. Explain its variation during the process of thermal interaction. **5**

4. (A) Attempt any **one** :–

(i) Consider a large box having area 'A' divided in k cells of areas $a_1, a_2, \ldots a_k$. N identical balls are thrown in the box in completely random manner. Find the condition for the resulting most probable distribtuion. **10**

(ii) What are Bosons? Derive Bose - Einstein distribution law.

(B) Attempt any **one** :– **10**

(i) State Planck's formula for black body radiation and hence obtain Wien's displacement law. **5**

(ii) Evaluae α and β in the Maxwell - Boltzmann distribution law given by equation : $n_i = g_i\, e^{-\alpha}\, e^{-\beta u_i}$ **5**

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5. (A) Attempt any **one** :–

(i) The equation of motion of a body falling under gravity in a resistive medium is 4

$$\frac{dv}{dt} + bv = g$$

solve this equation for v if the body starts from rest.

(ii) Solve $\frac{dN(t)}{dt} = -\lambda N(t)$ by using the method of seperation of variables. 4

(B) Attempt any **one** :–

(i) State Dirichlet's theorem. Explain the Dirichlet's conditions. 4

(ii) Get the Fourier transforms of first order and second order derivatives of a function f(x). 4

(C) Attempt any **one** :–

(i) Energy difference between two particle states in a system is 3.2×10^{-21} J. Calculate temperature of the system when ratio of the probabilities of the two states is e^2. 4

(ii) Consider a system of eight spin half particles fixed in uniform magnetic field B. If μ_0 is the magnetic moment associated with each particle, find the various possible macrostates of the system and statistical weight of each macrostate. Which will be the most probable macrostate? Explain. 4

(D) Attempt any **one** :–

(i) Calculate thermodynamic probability of the most probable distribution and least probable distribution of 1000 identical particles among 200 identical cells. 3

(ii) An enclosure of volume 100 cc is filled with black body radiation. Calculate the number of modes of vibration in the wavelength range 4000 AU to 4005 AU. 3
