Time: 3Hrs Marks:-100

N.B : (1) All questions are compulsory.
(2) Figures to the right indicate maximum marks.
(3) Use of non-programmable calculators is permitted.
(4) Symbols used have their usual meaning

Q1. A) Select correct answer (12)

1 The momentum operator in one dimension is

a) $-i\hbar\frac{d}{dx}$ b) $i\hbar\frac{d}{dx}$ c) $i\hbar\frac{d}{dt}$ d) $-i\hbar\frac{d}{dt}$

2 Which of the following is not a physical requirement for a wave function to be valid
a) Single valued b) continuous in given region c) time independent
d) None of these.

3 A particle is confined in a cubical box. The degeneracy of the energy state E, if E = $14\frac{h^2}{8mL^2}$ is

a) 6 b) 3 c) 9 d) 14

4 A particle of energy E approaches a potential step of height V, greater than E. According to quantum mechanics the particle is
a)always reflected b) always transmitted
c)may be reflected or transmitted d) None of these

5 α-particles are emitted from the nucleus by ______.
tunneling b) bombardment c) emission d) fission

6 Diatomic molecule is an example of ______.
a) harmonic oscillator b) simple oscillator
c) damped oscillator d) multiple oscillator

B) Answer in one sentence (3)

1 Give the statement of equation of continuity in classical mechanics with its usual meaning

2 What is tunnel effect?

3 What is energy of a simple harmonic oscillator in the lowest state known as?

C) Fill in the Blanks (5)

1 $|\Psi|^2 = ---$

2 ------ is the normalized condition for three dimensional wave function Ψ

3 The probability of finding the particle in classically forbidden region is called ……..

4 If particle is restricted to a limited region by external forces so that it moves back and forth in that region only, then energy states of the particle are called ……… states.

5 Scanning tunneling microscope (STM) type of microscope is based on the quantum mechanical phenomenon known as _______.

Q2.) Attempt any one (8)

1 How does de Broglie postulate enter into Schrodinger's theory?

2 Derive equation of continuity in quantum mechanics and discuss its significance.

B) Attempt any one (8)

1 Discuss Max Born interpretation of wave mechanics. Hence explain 'Normalization of wave function'.

2 Derive Schrodinger's Time Independent Equation (STIE).

C) Attempt any one (4)

1 Find the expectation value of particle position if the eigen function describing the particle is given by

$\Psi = ax$; $0 < x < 1$

$= 0$ elsewhere.

2 Show that for stationary states, expectation of momentum is independent of time (consider 1-D motion).

Q3. A) Attempt any one (8)

1 A particle is subjected to a three dimensional box and is subjected to a potential given by

$V(x) = 0$ inside the box

$V(x) = V_0$ outside the box

Write down Schrodinger's time independent wave equation and obtain normalised solution.

2 Consider an electron of energy **E** incident on the potential step defined by

$V(x) = 0$ for $x \leq 0$

$V(x) = V_0$ for $x \geq 0$

Show that the particle can penetrate into the second region even if its energy is less than V_0.

B) Attempt any one (8)

1 Set up Schrodinger's equation for a free particle. Solve the equation to obtain the eigenfunction. Show that the expectation value of momentum of the particle is same as the momentum that a classical particle will have.

2 Consider a particle confined to move in an infinite rectangular potential well. Show that expectation value of the position co-ordinate **x** of a particle in the well depends upon the length of the well.

C) Attempt any one (4)

1 A neutron of kinetic energy 5 MeV tries to enter a nucleus and its potential energy drops at the nuclear surface very rapidly from a constant external value V = 0 to a constant internal value V = – 50 MeV. Estimate the probability that the neutron will be reflected at the nuclear surface.

2 An α particle having energy 10 MeV approaches a potential step of height 50 MeV and width 10^{-15} m. Determine the transition coefficient if mass of α particle is $6{\cdot}68 \times 10^{-27}$ kg.

Q4. A) Attempt any one (8)

1 State correspondence principle. Show how quantum and classical probabilities of a one-dimensional oscillator leads to correspondence principle.

2 Discuss in detail the penetration of particle having energy E_0 across potential barrier of finite height V_0 and width (a) for the case $E_0 > V_0$.

B) Attempt any one (8)

1 Show that the STIE for a one-dimensional harmonic oscillator can be written in the form $(\frac{\partial^2}{\partial y^2} - y^2)\Psi = -2\varepsilon\Psi$

2 Establish the Schrodinger's equation for linear harmonic oscillator and solve it to obtain its eigen value and eigen function.

C) Attempt any one (4)

1 An α-particle having energy10 MeV approaches a potential barrier of height 30 MeV. Find the width of potential barrier if the transmission coefficient is 2×10^{-3}. (Given: mass of α-particle = 6.68×10^{-27}Kg).

2 A beam of electrons is incident on a potential barrier 5eV high and 5A wide. What should be their energy so that half of them tunnel through the barrier?

Q5. Attempt any Four (20)

1 Write a short on 'Operators

2 'Wave functions add, not the probabilities', explain '.

3 Show that the eigen functions of a quantum mechanical operator with different eigen values are orthogonal.

4 A particle arrives at a step potential having height V_0. Discuss the problem classically when energy of the particle is

(i) more than the step height

(ii) less than the step height

5 The wave function for the ground state of a harmonic oscillator of mass m and force constant k is proportional to $e^{-\frac{\alpha^2 x^2}{2}}$ where $\alpha^2 = \frac{m\omega}{\hbar}$ and $\omega^2 = \frac{k}{m}$. Show that this is a solution and find the corresponding eigen value

6 Find the expectation value <x> for the first excited state of a simple harmonic oscillator.

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