Time: 3Hrs **Marks:-100**

N.B : (1) All questions are compulsory.
(2) Figures to the right indicate maximum marks.
(3) Use of non-programmable calculators is permitted.
(4) Symbols used have their usual meaning

Q1. A) Select correct answer (12)

1 Hamiltonian operator is the
a) linear momentum operator. b) kinetic energy operator.
c) Total energy operator. d) angular momentum operator.

2 Eigen function of operator $\frac{d}{dx}$ with eigenvalue a is (A is Constant)
a) Ae^{ax} b) $Ae^{i\sqrt{ax}}$ c) $Ae^{\sqrt{ax}}$ d) Aae^{ax}.

3 The solution of Schrodinger wave equation are describe by stationary states when particle is moving in
a) time independent potential. b) time dependent potential.
c) velocity dependent potential. d) velocity independent potential.

4 Wave function ψ_n for a particle in one dimensional box has
a) (n +1) nodes b) n nodes c) (n – 1) nodes d) infinite nodes

5 Quantum mechanical tunneling is due to the
a) particle nature of matter. b) dual nature of matter.
c) wave nature of matter. d) random nature of matter.

6 Scanning tunneling microscope (STM) type of microscope is based on the quantum mechanical phenomenon known as
a)Tunneling effect. b) Ramsur-Townsend effect.
c) Correspondence principle. d) Ramsur effect.

B) Answer in one sentence (03)

1 What are operators?
2 What is degeneracy of energy states?
3 Give one example of harmonic oscillator.

C) Fill in the Blanks (5)

1 Steady state is when wave function representing the system is ------------ of time.
2 The Eigen functions are -------------------to each other.
3 The total probability of finding the particle in space must be ______.
4 A particle is confined to an infinite square well. The probability of locating it just outside the well is _______.
5 ______ is the ratio of the reflected probability current density to the incident probability current density.

Q2. A) Attempt any one (8)

1 Explain what is meant by expectation value of x. Why is it necessary to use the operator form of a physical quantity in calculating its expectation value? Why should the operator be sandwiched between $\psi *$ and ψ in the equation of expectation value?

2 Derive equation of continuity in quantum mechanics. State significance of probability flux.

B) Attempt any one (8)

1 State and explain the basic postulates of quantum mechanics.

2 Show that when the potential energy is a function of position alone, the Schrodinger time dependent equation reduces to Schrodinger time independent equation. What is meant by stationary states?

C) Attempt any one (4)

1 The eigen function for a free particle in a box is given by

$\Psi_n = A \sin \frac{n\pi x}{2L}$, n is an integer

Find its allowed energies.

2 For eigen function $\Psi(x) = \sqrt{\frac{2}{l}} \sin \frac{\pi x}{l}$; $0 < x < l$ determine $< x >$.

Q3. A) Attempt any one (8)

1 A beam of particles each of mass m and energy E, moving in a region of zero potential energy, approaches a step potential barrier of height V_0, where $E < V_0$. Obtain the reflection and transmission coefficients.

2 Solve the Schrodinger wave equation for a particle moving in an infinitely deep one dimensional potential well and obtain its energy levels. Draw the energy level diagram.

B) Attempt any one (8)

1 Solve the Schrodinger wave equation for a particle moving in a rectangular three dimensional box and obtain its energy levels. Obtain the total normalized wave-functions inside the box.

2 Set up Schrodinger's equation for free particle. Solve the equation to obtain the wave-function. Show that the wave function for the particle is an Eigen function of the linear momentum operator.

C) Attempt any one (4)

1 Find the probability of a particle trapped in a one dimensional box of length L can be found between 0.45 L and 0.55 L for the ground state.

2 Consider an atom as a cubical box of each side 10^{-10} m. Calculate the energy of an electron trapped in the atom in the ground state and the first excited state.

Q4. A) Attempt any one (8)

1 A particle of mass ***m*** and energy $E > V_0$ travelling along x-axis has a potential barrier described by

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \\ V_o & \text{for } 0 \leq x \leq a \\ 0 & \text{for } x > a \end{cases}$$

Write down the time-independent Schrodinger wave equation for the motion of particle, solve it and derive the expression for reflection and transmission coefficient of the particle.

2 Define Tunneling effect. Derive an expression of the approximate transmission coefficient of a potential barrier of height V_0 and width a.

B) Attempt any one (8)

1 Solve the linear harmonic oscillator equation and obtain an expression for energy eigenvalues.

2 State correspondence principle. Explain it with the help of probability density plot.

C) Attempt any one (4)

1 The potential barrier problem is a good approximation to the problem of an electron trapped inside but near the surface of a metal. Calculate the probability of transmission that a 2 eV electron will penetrate a potential barrier of 5 eV when the barrier width is 2 A°.

2 A harmonic oscillator consists of a mass 1 kg on a spring and it oscillates with amplitude 1 m and angular frequency 1 rad/s. What is the order of magnitude of the quantum number associated with the energy of the system and comment on the result?

Q5. Attempt any Four (20)

1 If $\Psi_1(x)$ and $\Psi_2(x)$ are normalized solutions of STIE for two different energy eigenvalues of a system, then is the following general solution $\Psi = 4\Psi_1 + 3\Psi_2$ normalized.
What is probability of locating the system in the state $\Psi 1$?

2 What is meant by superposition of wave functions? Show that wave functions obey the principle of superposition but the corresponding probability densities do not.

3 Obtain the Eigen-values of the momentum of the particle in a one dimensional box.

4 Show that the ground state energy level of a particle in one dimensional potential box with rigid walls is in agreement with uncertainty principle.

5 Show that for a simple harmonic oscillator $\frac{\Delta E}{E_n} = \frac{2}{2n+1}$.
What will happen to this ratio if $n \rightarrow \infty$?

6 Find the expectation value $< x >$ for the first excited state of a simple harmonic oscillator.
