QP CODE : 52352

3 HRS **100 MARKS**

Note: 1) All questions are compulsory.
2) Use of non- programmable calculator is allowed.
3) Draw figures wherever necessary.
4) Symbols have their usual meanings unless mentioned.

Q.1 (A) Select the correct option 12

i) The minimum number of non co-planar vectors, of unequal magnitudes which can be added so that their resultant is equal to zero, is
(a) 1 (b) 2 (c) 3 (d) 4

ii) Which of the vector fields is solenoidal?
(a) $x\hat{i}-z\hat{j}$ (b) $x\hat{i}-2y\hat{j}+\hat{k}$
(c) $3x\hat{i}-2y\hat{j}-z\hat{k}$ (d) $x\hat{i}-z\hat{j}$

iii) The differential equation $y''+xy'+x^2y^2=x^3$ is,
(a) Linear (b) Homogeneous (c) Second order (d) Second degree

iv) In an LCR circuit with a D.C. source, the voltage across the resistance may,
(a) Continuously Oscillate
(b) Be Constant in time
(c) Oscillate initially with decreasing amplitude
(d) Linearly increase with time

v) The periodic time of two oscillators are T and 5T/4 respectively. Both oscillators starts their oscillation simultaneously from the mid point of their path of motion. When the oscillator having periodic time T completes one oscillation, the phase difference between the two oscillators will be,
(a) 90^o (b) 112^o (c) 72^o (d) 45^o

vi) A metal wire having linear mass density 10 gm/m is passed over two supports separated by a distance of 0.25 m. The wire is kept in tension by suspending a 10 kg mass. The midpoint of the wire passes through a magnetic field provided by magnets and an a.c. supply having a certain frequency is passed through the wire. If the wire starts vibrating with its resonant frequency, what is the frequency of a.c. supply?($g = 10\ m/s^2$)
(a) 50Hz (b) 100Hz (c) 200Hz (d) 250Hz

(B) Answer in one sentence : 03

i) What vector does the gradient of a scalar function represent?
ii) How many linearly independent solutions can a linear second order inhomogeneous differential equation have?
iii) What is a dispersive medium?

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(C) Fill in the blanks.

i) If the scalar triple product of three vector is zero then we can say that the three vectors are _____.

ii) If $|\mathbf{A}+\mathbf{B}| = |\mathbf{A}|$, then $\mathbf{B}.(2\mathbf{A}+\mathbf{B})$ equals _____.

iii) If the characteristic equation for a linear second order homogeneous differential equation, with constant coefficients has two equal roots - say 'a' - , then the most general solution of the given differential equation will be of the form _____.

iv) When two mutually perpendicular simple harmonic motions having the same angular frequency and amplitude but having a phase difference of π^c are simultaneously superimposed the trajectory of the resulting motion will be a _____.

v) Waves given out by a point source of light have a _____wavefront.

Q.2 (A) Attempt any one. 08

i) $Find\ \vec{A}\,X\,(\vec{B}\,X\,\vec{C}),if\ \vec{A}=2\hat{\imath}-5\hat{\jmath}-\hat{k},\vec{B}=\hat{\imath}-5\hat{\jmath}+2\hat{k},\vec{C}=2\hat{\imath}+\hat{\jmath}-4\hat{k}$

ii) $Find\ \vec{A}.(\vec{B}\,X\,\vec{C}),if\ \vec{A}=2\hat{\imath}-5\hat{\jmath}-\hat{k},\vec{B}=\hat{\imath}-5\hat{\jmath}+2\hat{k},\vec{C}=2\hat{\imath}+\hat{\jmath}-4\hat{k}$

(B) Attempt any one 08

i) Find the derivative of $\phi=4xz^2-x^2y^2z$ at (2,-1,2) in the direction of vector $2\hat{\imath}-3\hat{\jmath}-6\hat{k}$

ii) $Show\ \vec{\nabla}.(\vec{\nabla}\,X\,\vec{A})=0,for\ \vec{A}=x^2y\hat{\imath}-y^2z\hat{\jmath}+z^2x\hat{k}$

(C) Attempt any one. 04

i) The diagonals of a parallelogram are

$\vec{A}=3\hat{\imath}-4\hat{\jmath}-\hat{k}\ and\ \vec{B}=2\hat{\imath}+3\hat{\jmath}-6\hat{k}$

Show that the parallelogram is a rhombus and find the length of its side.

ii) $Find\ \vec{\nabla}r^3,\vec{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$

Q.3 (A) Attempt any one. 08

i) Solve for y(x) given, $x(x^2 + 2y^2)dx + y(2x^2 + y^2)dy = 0$

ii) Obtain a solution for a general linear inhomogeneous first order differential equation.

(B) Attempt any one. 08

i) Solve the differential equation:-

$y'' + 3y' + 2y = e^{-x}$

ii) Solve:- $\ddot{y} + 2d\dot{y} + K^2y = 0\ (K > 0)$

Discus the solution for : i) d <K ii) d = K iii) d >K.

(C) Attempt any one. 04

i) Solve:- $y' = y^2$ for $y(0) = 1$

ii) Obtain the solution for the harmonic oscillator differential equation.

Q.4 (A) Attempt any one. 08

i) Obtain the trajectory of a particle on which two parallel simple harmonic motions having different angular frequencies are simultaneously superimposed.

ii) Show that a small disturbance on a string tied between two rigid supports, travels along the string as a wave. Hence deduce the speed of the wave?

(B) Attempt any one. 08

i) Explain normal modes of vibration of a string.

ii) Obtain an expression for the group velocity of a collection of waves traveling in a medium.

(C) Attempt any one. 04

i) The end of a string at z = 0 is driven harmonically at frequency 10 Hz and with amplitude 1cm. The velocity of the resulting wave is 5 meter/sec. What is the velocity of a point at a distance z = 3.25 m, when the velocity at z = 0 is zero?

ii) Check if the following functions are solutions of the wave equation:

$a)\ sin^2(x).\cos(3t)$

$b)\ \frac{1}{(5x-2t)^2}$

Q.5 Attempt any four 20

i) Find the angle between, $4\hat{i} - 2\hat{j} + 4\hat{k}$ & $3\hat{i} - 6\hat{j} - 2\hat{k}$

ii) $Given\ \emptyset(x,y,z) = x^2y^3z\ find\ \vec{\nabla}.(\vec{\nabla}\emptyset)$

iii) Derive an expression for current in a L-R circuit connected to a D.C. source.

iv) Check if the following differential equation is exact,

$$(4x^3 + 6xy + y^2)\frac{dx}{dy} = -(3x^2 + 2xy + 2)$$

v) What are Lissajous patterns?

vi) Find the trajectory of a particle subjected simultaneously to two mutually perpendicular S.H.M.'s given by,

a) $x = A\sin(\omega t)$ b) $y = B\sin(\omega t + \pi)$.

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