3 HRS 100 MARKS

Note: 1) All questions are compulsory.
2) Use of non- programmable calculator is allowed.
3) Draw figures wherever necessary.
4) Symbols have their usual meanings unless mentioned.

Q.1 (A) Select the correct option **12**

i) The quantities which require both magnitude and direction for their complete specification are called .....
(a) scalars (b) vectors
(c) polygons (d) None of the above

ii) The vector field V is irrotational if .....
(a) $\nabla \times V = 0$ (b) $\nabla \cdot V = 0$
(c) $V \cdot \nabla = 0$ (d) $V \times \nabla = 0$

iii) The degree of the equation $\left(\frac{d^3y}{dx^3}\right)^2 + x\frac{dy}{dx} = 2y$ is ---------.
(a) 0 (b) 3
(c) 1 (d) 2

iv) In the LR-series circuit has a steady emf E, which is switched on at a time t = 0. The current after a long time will be ----------.
(a) zero (b) $\frac{E}{\sqrt{R^2+L^2}}$
(c) $\frac{E}{L}$ (d) $\frac{E}{R}$

v) Energy is not carried by
(a) Transverse travelling waves (b) Stationary waves
(c) Longitudinal travelling waves (d) Electromagnetic waves

vi) In progressive wave the disturbance propagates with definite .....
(a) velocity (b) displacement
(c) acceleration (d) time

(B) Answer in one sentence : **03**

i) Define unit vectors.
ii) What is differential equation?
iii) Define transverse wave.

(C) Fill in the blanks **05**

i) If higher the frequency of wave, the shorter its ---------.
ii) Time interval during which the oscillation repeats itself is known as .....of oscillations.
iii) .....is the time constant of series LR circuit.
iv) The operator del is known as ---------.
v) For ..... medium group velocity is equal to phase velocity.

Q.2 (A) Attempt any one **08**

i) Define Scalar (DOT) product of two vectors. Write any six properties of Scalar (DOT) product of vectors.
ii) (a) If $\phi(x,y,z) = x^2 - y^2 + 2yz + 2z^2$, find $\nabla\phi$ at (1,-2,1)
(b) if $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\vec{k}$ is the position vector with respect to right handed co-ordinate system, find $\nabla \cdot \vec{r}$.

(B) Attempt any one **08**

i) If (a) $\phi = ln|\vec{r}|$ and (b) $\phi = \frac{1}{r}$, find $\overrightarrow{\nabla \phi}$.

ii) (a) Find the angle between $\overrightarrow{A} = 2\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\overrightarrow{B} = 6\,\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$.
(b) Prove that $\overrightarrow{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{C} \cdot \vec{A}$ (distributive law)

(C) Attempt any one **04**

i) Find the relation between $\overrightarrow{A}$ and $\overrightarrow{B}$ if $|\overrightarrow{A} + \overrightarrow{B}| = |\overrightarrow{A} - \overrightarrow{B}|$

ii) If $\phi = x^4 + y^4 + z^4$, determine $\nabla\phi$.

Q.3 (A) Attempt any one **08**

i) Discuss second order homogeneous linear ordinary differential equation with constant coefficients with roots of the equation are real and repeated (equal)

ii) A steady emf is suddenly applied to a circuit consisting of an inductance L and a resistance R in series. Obtain an expression for the growth of current in the circuit and also its decay when the emf withdrawn.

(B) Attempt any one **08**

i) (a) Solve the differential equation $\frac{dy}{dx} + y = e^{-x}$.

(b) Solve the differential equation $\frac{dy}{dx} = \frac{x^2}{e^y}$.

ii) Derive an expression for the growth of charge in the LCR series circuit. A steady emf is connected to the circuit consisting of a resistance R, an inductance L and a capacitor C in series.

(C) Attempt any one **04**

i) Solve the differential equation $\frac{dy}{dx} - kxy = 0$.

ii) A DC voltage of 100V is switched on to a circuit containing a resistance of 10Ω in series with an inductance of 10H. Find the rate of growth of current at the instant when the current is 5A.

Q.4 (A) Attempt any one **08**

i) Obtain an expression for the resultant motion when a particle is subjected simultaneously to two mutually perpendicular Simple Harmonic Motions of identical period and having same center. Give some special cases.

ii) Obtain the expression for the velocity of the transverse wave on a stretched string.

(B) Attempt any one **08**

i) Obtain an expression for the resultant motion when two Simple Harmonic Motions of same period and with same center along the same straight line act simultaneously on a particle. Give some special cases.

ii) Explain the terms phase velocity and group velocity.

(C) Attempt any one **04**

i) Show that the general solution of the wave equation, $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\left(\frac{\partial^2 y}{\partial t^2}\right)$ can be represented by y = $f_1(x - ct) + f_2(x + ct)$.

ii) Two tuning forks produce Lissajous figures. The figures changes in form of circular figure and after every 0.2 the circular figure occurs. If the frequency of one of the fork is 250 Hz. Find the possible frequencies of the other fork.

Q.5 Attempt any four **20**

i) Prove that $\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0$

ii) Show that the field $\vec{V} = cos\, y\, \hat{\imath} - xsin\, y\, \hat{\jmath} - \cos z\ \hat{k}$ is conservative.

iii) Define the exact differential equation.

iv) A coil of self-inductance 200 H is joined to the terminals of a battery of emf 5 volts through a resistance of 20 Ω. What is the time constant of the circuit? What is the maximum current that is finally established in the circuit?

v) Show that a particle whose potential energy is $\frac{1}{2} Kx^2$, where K is a constant, executes simple harmonic motion.

vi) Distinguish between progressive and stationary waves.