[Time: Three Hours] **[Marks:100]**

Please check whether you have got the right question paper.

N.B:
1. Figures to right indicate full marks.
2. Use of non-programmable calculator is permitted.

Q1 A Select the correct option:- **12**

i If two SHMs of the same amplitude, time period and phase act at right angles to each other, then the resultant vibration is ______
(a) elliptical (b) circular
(c) straight line (d) parabolic

ii. A wire fixed at one end is stretched horizontally and has a weight of 1 kg on the other end after passing it over a pulley. If a segment of 15 cm of standing vibration has a frequency 131 Hz, the mass per unit length of the wire is………..
(a) 0.036 gm/cm (b) 0.063 gm/cm
(c) 0.054 gm/cm (d) 0.045 gm/cm

iii. If $\bar{A} = \hat{i} - \hat{j} + \hat{k}$, $\bar{B} = \hat{i} + 2\hat{j} - \hat{k}$, then $\bar{A} . \bar{B}$ = ---------
a) 0 b) 2 c) -1 d) -2

iv. If $\hat{\varphi} = 3x^2 y \hat{i}$, $\bar{\nabla} . \hat{\varphi}$ at (1,-1,0) = -----
a) $3\hat{i} - \hat{j} + 4\hat{k}$ b) $-6\hat{i}$ c) $6\hat{i} + \hat{j} + 4\hat{k}$ d) $\hat{0}$

v. The degree of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$ ……….
a) 0 b) 1 c) 2 d) 4

vi. The order of differential equation $\frac{dN}{dt} = -\lambda N$ is --------------
a) 0 b) 1 c) 2 d) 3

B Answer in **ONE** statement **3**
i. Define group velocity.
ii. What do you mean by an ir-rotational vector?
iii. What is exact differential?

C Fill in the blanks : **5**

i. Wave is defined as ………disturbance propagated through a particular medium.
ii. The magnitude of maximum value of displacement on either side from the equilibrium position is called…….
iii. The position vector of point P(3,2,1)with respect to origin is ----
iv. A differential equation which contains only one variables is called………….differential equation.
v. A differential equation is said to be homogeneous when F(x) = …………

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Q2 A Attempt Any **ONE** : **8**

i Show that $\bar{A}$, $\bar{B}$ and $\bar{C}$ are mutually orthogonal unit vectors where:

$\bar{A} = \frac{2\hat{i}-2\hat{j}+\hat{k}}{3}$, $\bar{B} = \frac{\hat{i}+2\hat{j}+2\hat{k}}{3}$ and $\bar{C} = \frac{2\hat{i}+\hat{j}-2\hat{k}}{3}$

ii If $\bar{A} = 2\hat{i} + 4\hat{j} - 5\hat{k}$, $\bar{B} = -\hat{i} - \hat{j} + 3\hat{k}$, then examine whether $|\bar{A} + \bar{B}| = |\bar{A}| + |\bar{B}|$. Also find a vector of magnitude 7 and parallel to $|\bar{A} + \bar{B}|$.

B Attempt Any **ONE**: **8**

i Determine the constant a so that the following vector is solenoidal:
$\bar{V} = (-4x - 6y + 3z)\hat{i}$ + (-2x + y – 5z) $\hat{j}$ + (5x + 6y + az) $\hat{k}$. Hence find $\bar{\nabla} \times \bar{V}$.

ii Find the directional derivative of $\emptyset = 4xz^3 - 3x^2y^2z$ at (2, -1, 2) in the direction $2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$. In what direction will the directional derivative be maximum? Find the magnitude of the maximum.

C Attempt Any **ONE**: **4**

i Prove using vector method that the diagonals of a rhombus are mutually perpendicular.

ii If $\bar{A} = x^2z\,\hat{i} - 2\,y\,z^2\,\hat{j} + xy^2\hat{k}$, find div ($\bar{A}$) at (0,-1,1).

Q3 A Attempt any **ONE** : **8**

i. Show that the following differential equation is exact and hence find its solution:
$(x^2 + \ln y)\,dx + \frac{x}{y}\,dy = 0$

ii. Discuss second order homogeneous linear ordinary differential equations with constant co-efficients with roots of the equation real and distinct.

B Attempt any **ONE** : **8**

i. A small magnet is suspended by a vertical string so that it can rotate only in the horizontal plane, The relation between the small horizontal deflection θ of the magnet under the action of the earth's magnetic field is

$$I\,\frac{d^2\theta}{dt^2} + MH\theta = 0$$

Where I is the moment of inertia of the magnet about vertical axis, M is the magnetic moment of the magnet and H is the horizontal component of the earth's magnetic field. Solve the equation and find the time period of the oscillations.

ii. Discuss the general first order linear differential equation with reference to complementary function and particular integral. Obtain its general solution.

C Attempt any **ONE** : **4**

i. Solve the equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$

ii. Solve the equation $\frac{dy}{dx} + \frac{2}{x}y = \frac{x^2}{2}$.

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Q.4 A Attempt any **ONE:** **8**

i. Obtain an expression for the resultant motion when two SHMs of the same period and with same centre along the same straight line (two collinear SMHs of same period and centre) act simultaneously on a particle. Give some special cases.

ii. Deduce an expression for the speed of transverse wave along a stretched string.

B Attempt any **ONE:** **8**

i. Obtain an expression for the resultant motion when a particle is subjected simultaneously to two mutually perpendicular SHMs of identical period and having same centre. Give some special cases .

ii. Discuss the general solution of wave equation for one dimensional motion.

C Attempt any **ONE:** **4**

i. Show that the general solution of the wave equation,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$$

Can be represented by $y = f_1(x - ct) + f_2(x + ct)$

ii. Standing waves are produced by the superposition of two waves,

$y_1 = 10\sin(3\pi t - 4x)$ and
$y_2 = 10\sin(3\pi t + 4x)$

Find the amplitude of motion at x = 18.

Q5 Attempt Any **Four** : **20**

i Particle is influenced simultaneously by two collinear SHMs which are given by, $x_1 = 6\sin(\omega t + \pi/3)$ and $x_2 = 2\sin\omega t$. Find the equation of resultant motion.

ii Explain the terms: (a) Amplitude, (b) Phase, (c) Epoch, (d) Frequency, (e) Time period

iii. Find $\overline{\nabla}\varphi$ where $\varphi = (x^2 + y^2 + z^2).\, e^{-\sqrt{x^2 + y^2 + z^2}}$.

iv. Find the projection of the vector $\overline{A}$ on the resultant of vectors $\overline{B}$ and $\overline{C}$ where:
$\overline{A} = 2\hat{i} - 3\hat{j} + 6\hat{k}$, $\overline{B} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\overline{C} = -6\hat{i} + 2\hat{j} - 3\hat{k}$

v. Solve the following equation for θ.
$\frac{d\theta}{dx} = -k(\theta - \theta_0)$ where k is constant.

vi. In an LC circuit derive an expression for the charge on the capacitor. Initially both current & charge are zero.

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