

Note: i) All the questions are compulsory.

ii) Figures to the right indicate full marks.

iii) Use of non-programmable calculator is allowed.

Q.1) A) Attempt any one. [8M]

a) If $A = i - 2j - 3k$, $B = 2i + j - k$ & $C = i + 3j - 2k$.

Find (i) $|(A \times B) \times C|$ (ii) $(A \times B) \times (B \times C)$ (iii) $(A \times B) \cdot C$ (iv) $|A \times (B \times C)|$

b) If $A = 2yz \, i - x^2 y j + xz^2 k$, $B = x^2 i + yz j - xy k$ & $\phi = 2x^2 yz^3$.

Find (i) $(A \cdot \nabla)\phi$ (ii) $A \cdot \nabla \phi$ (iii) $(B \cdot \nabla)A$ (iv) $(A \times \nabla)\phi$.

Q.1) B) Attempt any one. [7M]

a) Find $\nabla \times A$ at point $(1, -1, 1)$. If $A = xz^3 \, i - 2x^2 y z j + 2yz^4 k$.

b) Show that the addition of vectors is associative, i.e. $A + (B + C) = (A + B) + C$.

Q.1) C) Attempt any one. [5M]

a) If $A = 2i + 7j + 2k$ and $B = 3i - j - 2k$, find (i) $A \cdot B$ (ii) $|A \cdot B|$ (iii) $(2A + 3B)$

b) Prove that $\nabla(F \cdot G) = G \nabla F + F \nabla G$

Q2 A) Attempt any one. [7M]

a) Solve the equation is exact and hence find its solution.

$$(x^2 + \ln y) dx + \frac{x}{y} dy = 0$$

b) Solve the equation $\frac{dy}{dx} + \frac{2}{x}y = \frac{x^2}{2}$

Q2 B) Attempt any one. [8M]

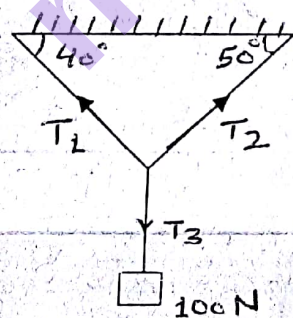
- a) The emf equation for the charging of a capacitor circuit is $R(dq/dt) + q/c = E$. Derive the expression for the charge on the capacitor. The initial charge on capacitor is zero.
- b) A body starts from rest and fall under gravity in a resistive medium. If we assume that the resistive force is proportional to the velocity its equation of motion is $(\frac{dv}{dt}) + bv = g$. Solve the equation for v .

Q2 C) Attempt any one. [5M]

- a) Show that $y = e^x + e^{2x}$ is a solution of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$
- b) $\ln y = -x(\ln x - 1)$ is solution of $\frac{dy}{dx} + \ln x^y = 0$

Q3 A) Attempt any one [8M]

- a) Derive Bernoulli's equation, What principle is it based? Is it equation applicable for a real liquid flow?
- b) A weight of 100N hangs from a string tied to the two other strings attached to a fixed support as shown in figure below. The two strings make angles of 40° and 50° with the horizontal. Find the tensions in all three strings.



Q3) B) Attempt any one. [7M]

a) For a homogeneous isotropic material show that $\sigma = \frac{3K-2\eta}{6K+2\eta}$.

b) The velocity above and below the wings of an airplane are v_1 and v_2 respectively and A is the area of the wings show that the upward force is $\frac{1}{2} A (v_1^2 - v_2^2) \rho$, Where ρ is the density of air.

Q3) C) Attempt any one [5M]

a) A thin metal plate of 10 cm^2 in area rests on a layer of oil 2mm thick A force of 0.01N applied to the plate horizontally keep it moving with a uniform speed of 1cm/find the viscosity of the oil.

b) For a steel material, $Y=2 \times 10^{11} \text{ N/m}^2$ and Bulk modulus is $13.3 \times 10^{10} \text{ N/m}^2$, calculate Poisson's ratio and modulus of rigidity of steel.

Q.4) Attempt any three. [15M]

a) (i) define scalar & vector.

(ii) Classify the following in scalar & vectors.

Charge, Kinetic Energy, Entropy, Frequency, Force, Magnetic field.

b) Prove that $(y^2 - z^2 + 3yz) \hat{i} + (3xz + 2xy) \hat{j} + (3xz + 2xz + 2z) \hat{k}$ is solenoid.

c) Show that the function $F(x,y)$, $F(x,y) = x^4 + 3x^2y + xy^2$ dF is a perfect differential?

d) Solve the equation $\frac{d^2y}{dx^2} + w^2y = 0$

e) Determine the force per unit area required to compress a volume of water by one percent. (Bulk modulus of water is $5 \times 10^{10} \text{ dynes/cm}^2$)

f) A thin metal plate of 20 cm^2 in area rests on a layer of oil 3mm thick A force of 0.01N applied to the plate horizontally keep it moving with a uniform speed of 2cm/s. Find the viscosity of the oil.