

Duration:[2½Hours]

[Total Marks: 75]

- N.B. 1) All questions are compulsory.
2) Figures to the right indicate full marks.

1. (a) Attempt any **ONE** question:

- i. If G is k -critical graph then show that
 - I) G is connected
 - II) Every vertex v of graph G has atleast $k - 1$ degree.
 - III) Graph G cannot be partitioned into subgraphs.
- ii. Prove that a graph G with $p \geq 2$ is 2-connected if and only if any two vertices are connected by at least two internally disjoint paths.

(8)

(b) Attempt any **TWO** questions:

(12)

- i. Show that vertex connectivity of a graph G is always less or equal to the edge connectivity of G .
- ii. Let $\pi_k(G)$ denote the chromatic polynomial of the graph G . If G is simple graph then prove that $\pi_k(G) = \pi_k(G - e) - \pi_k(G.e)$ where e is an edge of G .
- iii. Show that a connected graph G on p vertices is a tree if and only if the chromatic polynomial of G is $k(k - 1)^{p-1}$.
- iv. For any graph G , prove that $\chi(G) \leq \Delta(G) + 1$ where $\chi(G)$ represents vertex chromatic number of a graph G and $\Delta(G)$ denotes the maximum degree of G . Give an example of graphs for which $\chi(G) > \Delta(G)$.

2. (a) Attempt any **ONE** question:

(8)

- i. State and prove Max Flow - Min Cut Theorem.
- ii. Show that there are exactly five regular polyhedra.

(b) Attempt any **TWO** questions:

(12)

- i. State and prove Euler theorem for planar graph.
- ii. Show that if G is a planar (p, q) graph in which every face is bounded by a cycle of length at least n then show that $q \leq \frac{n(p-2)}{n-2}$.
- iii. If G is a graph with p vertices and q edges, and let every vertex of G has degree at least 6 then prove that $q \geq 3p$. Hence prove that every planar graph contains a vertex of degree at most 5.
- iv. Show that the complete graph K_5 and complete bipartite graph $K_{3,3}$ are nonplanar.

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3. (a) Attempt any **ONE** question: (8)
- State and prove Hall's (Marriage) Theorem for a System of Distinct Representatives.
 - An elf has a staircase of n stairs to climb. Each step it takes can cover either one stair or two stairs. Find a recurrence relation for a_n , the number of different ways for the elf to ascend the n -stair staircase and solve it by using generating function.
- (b) Attempt any **TWO** questions: (12)
- Let B denotes a forbidden chess board in which a special square $*$ has been identified and let D denote the board obtained from the original board by deleting the row and column containing the special square and E denote the board obtained from the original board where only the special square $*$ is removed from the board, then prove that $R(x, B) = xR(x, D) + R(x, E)$.
 - Show that a matching M in G is a maximum matching if and only if G contains no M -augmenting path.
 - Show that the number of non-negative integer solutions of the equation $x_1 + x_2 + \cdots + x_k = r$ is given by $\binom{r+k-1}{r}$.
 - Solve recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$, ($n \geq 2$) subject to initial conditions with $a_0 = 1, a_1 = -2$ using generating function.
4. Attempt any **THREE** questions: (15)
- Determine the chromatic polynomial and chromatic number of a graph G obtained by deleting an edge from K_4 .
 - If G is a bipartite graph, then show that $\chi'(G) = \Delta(G)$, where $\chi(G)$ represents vertex chromatic number of a graph G and $\Delta(G)$ denotes the maximum degree of G .
 - Prove that for any flow f and any cut (S, \bar{S}) , $val(f) = f^+(S) - f^-(S)$.
 - Show that the edge e is a loop in G if and only if e^* is a bridge in G^* .
 - Let h_n denote the number of non-negative integral solutions of the equation $3e_1 + 4e_2 + 2e_3 + 5e_4 = n$. Find the generating function $f(x)$ for $h_0, h_1, \dots, h_n, \dots$
 - Find the rook polynomial for the following $\{(1, 1), (2, 5), (3, 3), (4, 2), (4, 4), (5, 1), (5, 3)\}$.
