Q. P. Code: 05013

(REVISED COURSE)

Duration:[2½Hours] **[Total Marks: 75]**

N.B. 1) All questions are compulsory.
2) Figures to the right indcate full marks.

1. (a) Attempt any **ONE** question: (8)

   i. For a simple graph G of order p and size q, prove that $\pi_k(G)$, the chromatic polynomial of the graph G, is monic polynomial of degree p in k with integer coefficients and constant term zero. Further prove that its coefficients are alternate in sign and the coefficient of k^{p-1} is $-q$.

   ii. Prove that a graph G with $p \geq 2$ is 2-connected if and only if any two vertices are connected by at least two internally disjoint paths.

   (b) Attempt any **TWO** questions: (12)

   i. Show that vertex connectivity of a graph G is always less or equal to the edge connectivity of G.

   ii. Show that a connected graph G on n vertices is a tree if and only if the chromatic polynomial of G is $k(k-1)^{n-1}$.

   iii. Lel $\pi_k(G)$ denote the chromatic polynomial of the graph G. If G is simple graph then prove that $\pi_k(G) = \pi_k(G-e) - \pi_k(G.e)$ where e is an edge of G.

   iv. Show that every tree with $n \geq 2$ vertices is 2-chromatic. Is converse true? Justify.

2. (a) Attempt any **ONE** question: (8)

   i. Show that every planar graph is 5 vertex colorable.

   ii. Show that there are exactly five regular polyhedra.

   (b) Attempt any **TWO** questions: (12)

   i. Show that if G is a planar (p, q) graph in which every face is bounded by a cycle of length at least n then show that $q \leq \frac{n(p-2)}{n-2}$.

   ii. Let f be a flow in a network N and P be any f-incrementing path then show that there exist a revised flow f' such that $val(f') = val(f) + \epsilon(p)$

   iii. Show that edges in a plane graph G form a cycle in G if and only if the corresponding dual edges form a bond in G^*.

   iv. Show that if G is a planar graph in which degree of each face is 3, then $q(G) = 3p-6$.

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3. (a) Attempt any **ONE** question: (8)

   i. An elf has a staircase of n stairs to climb. Each step it takes can cover either one stair or two stairs. Find a recurrence relation for a_n, the number of different ways for the elf to ascend the $n-$stair staircase and solve it by using generating function.

   ii. State and prove the necessary and sufficient condition for a family of n sets to have System of Distinct Representative

   (b) Attempt any **TWO** questions: (12)

   i. If $\{A_1, A_2, ...A_n\}$ be a family of set, then prove that the largest number of sets of the family which together have a system of distinct representative equals the minimum value of expression $|A_{i_1} \cup A_{i_2} \cup \cdots \cup A_{i_k}| + (n-k)$ for all choices of $k = 1, 2, \ldots n$ and all choices of $i_1, i_2, \ldots, i_k$ with $1 \leq i_1 < i_2 < ... < i_k \leq n$.

   ii. Let $R_{n,m}(x)$ be the rook polynomial for the $n \times m$ chess board, all squares may have rooks. Show that $R_{n,m}(x) = R_{n-1,m}(x) + mxR_{n-1,m-1}(x)$

   iii. Solve recurrence relation $a_n = 2a_{n-1} + a_{n-2}$ for all $n \geq 2$ subject to initial conditions $a_0 = 3, a_1 = -2$ using generating function.

   iv. Find the coefficient of x^{16} in $(x^2 + x^3 + x^4 + \ldots)^5$. What is the coefficient of x^r?

4. Attempt any **THREE** questions: (15)

   (a) Let G be the graph with n vertices. Show that $\chi(G) \geq \frac{n}{n-\delta(G)}$ where $\chi(G)$ denotes vertex chromatic number of G and $\delta(G)$ denotes minumum degree of G.

   (b) Determine the chromatic polynomial and chromatic number of a graph G obtained by deleting an edge from K_4.

   (c) If G is planar graph with n vertices, m edges, f regions and k components then prove that $n - m + f = k + 1$.

   (d) If f is any flow and K be any cut in a network N then show that $val(f) \leq cap(K)$.

   (e) Show that a matching M in G is a maximum matching if and only if G contains no $M-$augmenting path.

   (f) Find the rook polynomial for the following $\{(1,1), (2,5), (3,3), (4,2), (4,4), (5,1), (5,3)\}$.