Duration:[3 Hours] **[Total Marks: 100]**

N.B. 1) All questions are compulsory.
2) Figures to the right indicate full marks.

1. Choose correct alternative in each of the following: (20)

i. Which of the following statement is true?
(a) In a simple graph there are at most two vertices of equal degree
(b) In a simple graph there at least two vertices of equal degree
(c) Degree of vertices are all distinct
(d) None of these

ii. The number of edges of Complete Bipartite graph $K_{m,n}$ is
(a) $m + n$ (b) mn (c) $m^2 + n^2$ (d) None of these

iii. Adjacency matrix of a simple graph G is
(a) Binary and symmetric (b) Only binary (c) Only Symmetric (d) None of these

iv. Let G be a graph on 8 vertices that has no pair of adjacent edges. The degree sequence of G is
(a) 2,2,2,2,1,1,1,1; (b) 1,1,1,1,1,1,1,1;
(c) 2,2,1,1,1,1,1,1; (d) such a graph does not exist.

v. A $G(p, p-1)$ graph is a tree if
(a) G is connected (b) G is acyclic
(c) Must be (a) or (b) (d) None of the above

vi. Let T be a tree with n vertices, where $n \geq 4$ and let v be a vertex of maximum degree in T, then T is a path if $d(v)$ is
(a) 3 (b) 2 (c) 1 (d) $n-1$

vii. If v is a cut vertex of a graph G, then
(a) G is Hamiltonian and Eulerian. (b) G is Hamiltonian but not Eulerian
(c) G is not Hamiltonian. (d) None of the above.

viii. Complete bipartite graph is Eulerian if
(a) $|V_1| = |V_2|$ (b) $|V_1| > |V_2|$
(c) $|V_1| = |V_2|$ = Even Number (d) None of these

ix. If G is Hamiltonian and if S is any non empty proper subset of $V(G)$ then
(a) $\omega(G-S) = |S|$ (b) $\omega(G-S) \leq |S|$
(c) $\omega(G-S) \geq |S|$ (d) None of these

x. Number of edges in cube graph Q_4 is
(a) 24 (b) 16 (c) 18 (d) 32

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2. (a) Attempt any **ONE** question from the following: (8)
   i. State and prove $Havel - Hakimi$ theorem for degree sequence of a graph G.
   ii. If $(A^n) = (a_{ij}^n)$ is the n^{th} power of adjacency matrix A of a graph G with $V(G) = \{v_1, v_2, \ldots v_n\}$, then prove that
   (a) a_{ij}^2, $i \neq j$ is the number of $v_i - v_j$ path of length 2.
   (b) $a_{ii}^2 = deg(v_i)$
   (c) $\frac{1}{6}$trace of A^3 is the number of triangles in G.

   (b) Attempt any **TWO** questions from the following: (12)
   i. State and prove Hand-Shaking theorem for Graph. Hence prove that every graph has an even number of odd vertices
   ii. For any graph G with at least 6 vertices, prove that either G or G^c contains a triangle.
   iii. If G is simple graph with p vertices, q edges and k components, then prove that $q \geq p - k$.
   iv. If G is graph of order p with $\delta(G) \geq (p-1)/2$, then show that G is connected. Is the bound $(p-1)/2$ sharp?, that is , in this case , can $(p-1)/2$ be replaced by$(p-2)/2$?

3. (a) Attempt any **ONE** question from the following: (8)
   i. Let G be a (p, q) graph. Prove that following statements are equivalent.
   a) G is tree.
   b) G is acyclic and $q = p - 1$.
   c) G is connected and $q = p - 1$.
   ii. State and prove Cayley's formula for spanning trees.

   (b) Attempt any **TWO** questions from the following: (12)
   i. Show that there exist a tree with degree sequence $d_1 \geq d_2 \geq ..... \geq d_n$ if and only if a) $d_i \geq 1$, for $1 \leq i \leq n$ and b) $\sum_{i=1}^{n} d_i = 2n - 2$
   ii. Define a cut vertex of a graph G. Show that vertex v is a cut vertex if and only if there exists two vertices x and y such that v is on every $x - y$ path in G.
   iii. Explain and write Huffman's algorithm for prefix code.
   iv. Write Breadth First Search and Depth First Search Algorithm for finding a spanning tree.

4. (a) Attempt any **ONE** question from the following: (8)
   i. Show that a nontrivial connected graph G is Eulerian if and only if every vertex of G has even degree.
   ii. If G is graph on p vertices with $p \geq 3$ and $\delta(G) \geq \frac{P}{2}$ where $\delta(G)$ denotes the minimum degree of G then show that G contains a Hamiltonian cycle.

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(b) Attempt any **TWO** questions from the following: (12)

i. Define a line graph $L(G)$ of a graph G. Show that if the simple graphs G_1 and G_2 are isomorphic, then its line graphs $L(G_1)$ and $L(G_2)$ are isomorphic. Is converse true? Justify.

ii. Let G be a simple graph with $p \geq 3$. If closure of G is complete, show that G is Hamiltonian.

iii. Prove that the cube graph Q_k is bipartite $k-$regular graph with 2^k vertices.

iv. If G is a (p, q) graph with $p \geq 3$ and $q \geq \frac{1}{2}(p-1)(p-2)+2$, then prove that G is Hamiltonian.

5. Attempt any **FOUR** questions from the following: (20)

(a) Show that a graph G is disconnected if and only if its vertex set V can be partitioned into two subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in the subset V_1 and the other in the subset V_2.

(b) Show that every $u-v$ walk W contains $u-v$ path.

(c) Prove that a connected graph G is a tree if and only if every edge of G is a cut edge.

(d) Show that every two vertices of a tree are connected by a unique path.

(e) If G is Hamiltonian graph then for every nonempty proper subset S of $V(G)$, prove that $\omega(G-S) \leq |S|$. Is converse true? Justify.

(f) Describe Fluery's Algorithm to find a closed Eulerian trail.

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