Q.P. Code: 40887

Time: 2:30 Hours   Total Marks: 75

N.B.
1. All questions are compulsory
2. Attempt any one from part (a) and any two from part (b) in each of the Questions 1, 2, 3.
3. Attempt any three from the Question 4.
4. Figures to the right indicate marks.

Q. 1 a i) State and prove Fubini's Theorem for a rectangular domain in $\mathbb{R}^2$. 8

ii) Define the triple integral of a bounded function $f : R \to \mathbb{R}$ where $R = [a,b] \times [c,d] \times [e,f]$ with usual notations prove that $m(b-a)(d-c)(f-e) \le \iiint_R f\, dV \le M(b-a)(d-c)(f-e)$. 8

b i) Evaluate $\int_0^1 \int_{-\sqrt{x-x^2}}^{\sqrt{x-x^2}} (x^2+y^2)\, dydx$ by converting to polar co-ordinates. 6

ii) Evaluate $\iint_S (x^2+y^2)\, dxdy$ by a suitable change of variables. where S is the region in the XY-plane bounded by the curves $x^2-y^2=1,\ x^2-y^2=2, xy=2, xy=4$. 6

iii) Evaluate the integrals $\int_0^2 \int_{1+y^2}^5 ye^{(x-1)^2}\, dxdy$ 6

iv) Verify the Fubini's theorem for $\iint_S f$ where $f(x,y)=x^2y$ and S is bounded by $x=2, x=4, x=2y$ and $x=y^2$. 6

Q.2 a i) When do you say that two parameterized curves in $\mathbb{R}^n$ are orientably equivalent? Define the line integral of a vector field F along an oriented curve Γ in an open set $U \subseteq \mathbb{R}^n$. If Γ and Γ' are two orientably equivalent curves in U, show that $\int_\Gamma F = \int_{\Gamma'} F$. 8

ii) State and prove Green's Theorem for a rectangle. 8

b i) Verify the Green's Theorem for $P(x,y)=2x-y+4$, $Q(x,y)=5y+3x-6$ and C is the triangle with vertices $(0,0),(3,0)$ and $(3,2)$ having positively oriented boundary. 6

ii) In the following problem show that the given line integral is independent of the path and Evaluate the line integral $\int_{(-1,2)}^{(3,1)} (y^2+2xy)dx + (x^2+2xy)\, dy$ 6

iii) Calculate the work done by the force field $F(x,y,z) = xi + yj$ when a particle is moved along the path $(3t^2,\ t,\ 1)\,;\ 0 \le t \le 1$. 6

iv) Evaluate the line integral of the vector field $F\,(x,y,z) = (x,^2 - xy\,,1)$ along the parabola $z=x^2,\ y=0,$ between $(-1,0,1)$ and $(1,0,1)$ 6

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Q.4 a i) For the surface $\bar{r}(u,v)$ described by the vector equation $\bar{r}(u,v) = X(u,v)i + Y(u,v)j + Z(u,v)\hat{k},\ (u,v) \in T$ where X, Y, Z are differentiable on T, define the fundamental vector product . If C is a smooth curve lying on the surface, $C = \bar{r}(\propto(t)),\ \propto; [a,b] \to T,$ then show that $\frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial u}$ is normal to C at each point. 8

ii) State and Prove Divergence Theorem for a simple solid region V bounded by an orientable surface S which can be projected on XY, YZ, ZX planes. 8

b i) Find the equation of the tangent plane at the point (2, 3, 0) to the surface which is parametrically given by $r(u,v) = (u+v)\bar{\imath} + 3u^2\bar{\jmath} + (u-v)\bar{k}$ 6

ii) Evaluate $\iint_S\ ydS$ where S is the part of the plane $3x + 2y + z = 6$ in the first octant. 6

iii) Use Stoke's theorem to evaluate $\oint_C\ F.d\bar{r},$ where $F(x,y,z) = xz\bar{\imath} + 2xy\bar{\jmath} + 3xy\bar{k}$ and C is the boundary of the part of the plane $3x + y + z = 3$ in the first octant 6

iv) Verify the divergence theorem over the sphere $x^2 + y^2 + z^2 = 9$ for $F(x,y,z) = (yz,\ xz,\ xy)$ 6

Q.4 i) Using double integration, find the area of the region S in $\mathbb{R}^2$ S is bounded by the parabola $y = x^2$ and line $y = 2x + 3$. 5

ii) Evaluate $\iiint_S\ xyzdV$ where S is the bounded by the three co-ordinate planes and the plane $x + y + z = 1$. 5

iii) Evaluate the integral of the scalar field $f(x,y,z) = x^2 + y^2 + z^2$ along the path $x(t) = \cos\ t,\ y(t) = \sin\ t,\ z(t) = t\ ;\ 0 \le t \le \pi/2$ 5

iv) Find whether the following force field F is conservative .If so find a scalar field $\emptyset$ so that $F = \nabla\emptyset$ and calculate the work done in moving the particle from the point P to the point Q where $F(x,y,z) = (y+z)\bar{\imath} + (x+z)\bar{\jmath} + (x+y)\bar{k}\ ;\ P(1,-1,0),\ Q(2,0,1)$ 5

v) Prove the following identities, assuming S and V satisfy the conditions of the Divergence Theorem and components of $\bar{F}$ have continuous partial derivatives, $\hat{n}$ is unit outward normal 5

a) $|V| = \frac{1}{3}\iint_S\ \bar{r}.\ \hat{n}dS$ where $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $|V|$ = volume of V.

b) $\iint_S\ curlF\ .\hat{n}dS = 0.$

vi) Find the surface area of the sphere $x^2 + y^2 + z^2 = a^2$. 5
